Okun's Law Testing Using Modern Statistical Data

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Description of the model

Okun's law is an empirically observed relationship relating unemployment to losses in a country's production first quantified by Arthur M. Okun. The "gap version" states that for every 1% increase in the unemployment rate, a country's GDP will be at an additional roughly 2% lower than its potential GDP. The “difference version” describes the relationship between quarterly changes in unemployment and quarterly changes in real GDP. The accuracy of the law has been disputed.

Okun's law is more accurately called "Okun's rule of thumb" because it is primarily an empirical observation rather than a result derived from theory. Okun's law is approximate because factors other than employment (such as productivity) affect output. In Okun's original statement of his law, a 3% increase in output corresponds to a 1% decline in the rate of unemployment; a 0.5% increase in labor force participation; a 0.5% increase in hours worked per employee; and a 1 % increase in output per hours worked (labor productivity).

Okun's law states that a one point increase in the unemployment rate is associated with two percentage points of negative growth in real GDP. The relationship varies depending on the country and time period under consideration.

The relationship has been tested by regressing GDP or GNP growth on change in the unemployment rate. Martin Prachowny estimated about a 3% decrease in output for every 1% increase in the unemployment rate (Prachowny 1993). The magnitude of the decrease seems to be declining over time in the United States. According to Andrew Abel and Ben Bemanke, estimates based on data from more recent years give about a 2% decrease in output for every 1% increase in unemployment (Abel and Bernanke, 2005).

There are several reasons why GDP may increase or decrease more rapidly than unemployment decreases or increases. As unemployment increases:

- a reduction in the multiplier effect created by the circulation of money from employees
- unemployed persons may drop out of the labor force (stop seeking work), after which they are no longer counted in unemployment statistics
- employed workers may work shorter hours
- labor productivity may decrease, perhaps because employers retain more workers than they need

One implication of Okun's law is that an increase in labor productivity or an increase in the size of the labor force can mean that real net output grows without net unemployment rates falling (the phenomenon of "jobless growth").

Arthur Okun estimated the following relationship between the two:

$$Y_t = -0.4 (X_t - 2.5)$$

This can also be expressed as a more traditional linear regression as:

$$Y_t = 1 - 0.4 X_t$$

Where Y_t is the change in the unemployment rate in percentage points. X_t is the percentage growth rate in real output, as measured by real GNP.

So we will be estimating the model:

$$Y_t = b_1 + b_2 X_t$$

Where Y_t is the change in the unemployment rate in percentage points. X_t is the change in the percentage growth rate in real output, as measured by real GNP. b_1 and b_2 are the parameters we are trying to estimate.

Required data for the estimation

In order to analyze and test the Okun's law, we should find out some specific data: a percentage change in GNP relative to previous quarter, which we will denote y_t, and change in the unemployment rate from last quarter, which we will denote as x_t.

We will take quarterly data from the 01.04.1948 to 01.07.2002.

Also it's important to mention, that the last interval of the 01.10.2002 shouldn't be involved in our analyzing right now, we will need it later for model forecasting.

For the evaluation of this model we are going to use data that was collected by U.S. government agencies and thus is in the public domain (https://fedstats.gov)

The steps of an econometric model testing.

Model Specification

Here is a mathematical interpretation of the Okun's law.

General form of the fitted line:

$$Y_i = \beta_0 + \beta_1 X_i + U_i ,$$

$$E_{u(t)} = 0, \quad \sigma_{u(t)} = const$$

Where Y_t – is the change in the unemployment rate in percentage points., X_{1t} – is the percentage growth rate in real output, as measured by real GNP, $\beta_{0,1}$ – parameters (sensitivity of the explained variable to changes of the explainable variable), U_i – the disturbance term.

Now let's turn to the precise explanation of all the steps in the Excel. Let us input the values of endogenous and exogenous variables from 1947-01-01 to 2002-07-01 into corresponding rows in the Regression, Analysis ToolPak.

We did not take data of 2002 year, because We are going to use it later when checking model adequacy. Level of significance is 95%.

Results of the regression statistics:

Table 1

Regression Statistics	
Multiple R	0,711246
R Square	0,505870
Adjusted R Square	0,503583
Standard Error	0,725891
Observations	218

The results of variance analysis are generated in Table 3, which are used to the coefficient of determination R^2.

Table 2

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	116,5189	116,5189	221,1327	0,00
Residual	216	113,8143	0,5269		
Total	217	230,3333			

Table 3 contains values of regression coefficients with a_i (95% - in our case) confidence probability and their statistical assessment.

Table 3

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Y interception	0,85	0,049	17,43	0,01	0,76	0,95
X_1	-1,82	0,122	14,87	0,01	-2,06	-1,58

Table 4 contains theoretical values $\hat{y}_i$, computed with the help of regression formula, and residual values.

Residual values are calculated as the difference between empirical y_i and theoretical $\hat{y}_i$.

Table 4

Year of observation	***Predicted Y***	***Residuals***
1948-04-01	0,979	0,605
…	…	…
2002-07-01	0,979	0,100

Specification of estimated econometric model

The specification of our model with calculated parameters:
Specification of Estimated model

$$\begin{cases} Y_t = \underset{(0,049)}{0,86} - \underset{(0,12)}{1,82} \cdot X_t + \underset{(0,72)}{u_t} \\ F = 221,13; \quad F_{crit} = 3,88; \\ R^2 = 0,506; \; t_{crit} = 3,336 \end{cases}$$

Where β_0 = 0,857 with standard error of 0,04917, β_1 = (-1,826) with standard error of 0,12282, the standard error of disturbance term is 0,72589.

Model testing

Now let's move to another important step – model testing. The calculated regression coefficients β_i allow us to construct an equation of Okun's law.

The equation is: Y_t= 0,857 -1,826Xt + ε_t, where ε_t is random value.

R^2 Coefficient

Value of multiple coefficient of determination R^2 equals to 0.506 shows that 50.6% of total deviation of y_i is explained by the variation of the factors x_i. Honestly speaking, such value of the R^2 is not good, as it's not so near to 1. It means that selected factors don't' effect significantly to our model, which confirms the correctness of their inclusion in the estimated model.

Significance F

The calculated level of significance 0.0000001<0.05 (*Significance F*, table 2) confirms the R^2 significance.

F-test

Another way of checking R^2 is based on testifying whether F (table 2) is within the interval (F_{crit};$+\infty$) or not. In our case F_{crit} =Fраспобр(0.05;2;216)= 3,038, where 2 is the number of degrees of freedom, it equals to the number of the equation regresses m=2, and 216 is the number of degrees of freedom, it equals to n-(m+1).

As our F=221,1327 and it is within the interval (3,038;$+\infty$), the H_0 hypothesis that R^2 =0 is rejected. That means coefficient of determination R^2 is thought to be significant.

Standard Error

Now we are going to test the importance of regression coefficients β_i.

Comparing the elements of the columns *Coefficients* and *Standard Error* (Table 3), we can notice that absolute values of standard errors is less than the corresponding values of coefficients, therefore, at the first stage of analysis, all the variables should remain in the model.

t-test

Let us check the significance of these coefficients with the help of t-test.

That is to test the inequality $|t| \le t_{crit}$, where t is the value of t-statistics (*t Stat*, table 3). If the inequality is right, the coefficient and its factor variable are considered to be non-significant; in the other case the coefficient and the regressor are considered to be significant.

In our case the critical value can be calculated using the special tables. For the considered example, value t_{crit}=1,971, where 0.05 is the level of significance, 216 – number of observations, 1 – number of factors in the regression equation, 1 – number of free terms in the regression equation.

All absolute values of t-statistics in table 3 are more than t_{crit}, therefore, all the regression coefficients are significant.

P-value test

Another common way of regression coefficients significance testing is based on P-value indicator implementation (*P-value*, table 3).

All p-values are less than our level of significance (α=0.05). Thus, all coefficients are being significant.

After all above-listed tests, we can conclude about a good model specification and the significance of the coefficients.

Goldfield-Quandt test

Goldfield-Quandt test is designed to check the second assumption of Gauss-Markov theorem about homoscedasticity of random disturbances, i.e. about the following equality satisfying:

$$Var(\varepsilon_1) = Var(\varepsilon_2) = ... = Var(\varepsilon_n) = \sigma^2$$

To implement it in the situation of the model of a linear multiple regression, it is necessary to:

1. Sort the initial data in ascending of the regressors sums of absolute values $(x_{1t}, x_{2t}, ..., x_{mt})$;

2. Split ordered data into two arrays so as (m+1)<k<n/2, where k – number of observations in the first array, m- number of regressors, m+1 – number of explainable regression function coefficients;

3. Assess a regression equation for each array separately, using Regression, Analysis ToolPak. This results in two models with the same equations, but with different coefficients;

4. Calculate Goldfield-Quandt GQ statistics as the ratio of the sum of squares of deviations of empirical data to the theoretical one for the first array (RSS_1) to the corresponding amount, calculated for the second array (RSS_2), $GQ = RSS_1 / RSS_2$. RSS can be found in *ANOVA* table, *SS* column, *Residual* raw;

5. Evaluate the critical value F_{crit} for given significance level α with ν_1 and ν_2 degrees of freedom, where $\nu_1 = \nu_2 = k-(m+1)$, k – number of observations in the first array, m – number of regressors. Fраспобр(α;ν_1;ν_2);

6. Second assumption about homoscedasticity of random disturbances is thought to be adequate if both of the following inequalities are valid: $\begin{cases} GQ \le F_{crit} \\ 1/GQ \le F_{crit} \end{cases}$

Otherwise, conclude about heteroscedasticity of random disturbances. This leads to loss of unbiased property of estimation of the parameters of the linear regression model obtained by the method of least squares, and the accuracy inadequacy of the characteristics of these estimations.

Let us consider our case. We should split ordered by the sum of x_{it} data into two arrays. The first one contains 109 observations, while the second = 108. Using Regression, Analysis ToolPak we can see the values of $RSS_{1,2}$ and, hence, estimate GQ, $1/GQ$, $F_{crit\ GQ}$ (table 5). $F_{crit\ GQ}$ is calculated by function Fраспобр(α;ν_1;ν_2), where α=0.05, $\nu_1=\nu_2$=k-(m+1)=107, where k – number of observations in the first array, m – number of factors. Thus, both inequalities are valid. Assumption about homoscedasticity of random disturbances is adequate.

Goldfield-Quandt test Table 5

RSS_1	55,70
RSS_2	54,70
GQ	1,018
1/GQ	0,981
$F_{crit\ GQ}$	1,828

Durbin-Watson test

This test is designed to check a particular case of third assumption of the Gauss-Markov theorem about the absence of autocorrelation between adjacent random residuals in the model.

$Cov(\varepsilon_i, \varepsilon_j) = 0$ if $j = i - 1$

Using values of the residuals ε_t, we can compute Durbin-Watson statistics: $DW = \sum_{t=2}^{n}(\varepsilon_t - \varepsilon_{t-1})^2 \Big/ \sum_{t=1}^{n}\varepsilon_t^2$

Then, we should find Durbin-Watson statistics critical values d_L and d_U with the help of special statistical table, where n=217 – total number of observations, k=1 – total number of factors.

There are three possible outcomes of the test:

- $d \in \{(0;d_L);(4-d_L;4)\}$ => positive/negative autocorrelation of the model's residuals exists;
- $d \in \{(d_L;d_U);(4-d_U;4-d_L)\}$ => autocorrelation of the model's residuals is ambiguous;
- $d \in (d_U;4-d_U)$ => positive/negative autocorrelation of the model's residuals does not exist.

Confidence interval

We should estimate the lower and upper boundaries for each year. We will use the following formula: 99,9% (I had to decrease the level of significance as with the significance level of 95% not all of our data laid in our intervals) boundary=$\hat{Y}_i +/- t_{criti} * st.error$, where t_{crit} is calculated as it has been shown in part "Model testing", section "t-test" and standard error = 0,725891 (Standard error of new Specification, table 1), $\hat{Y}$ - predicted value of y_t (Predicted Y according to new specification). Then we should compare the empirical data for each year with resulted interval boundaries (Appendix, Table 8).

Adequacy checking

Let us check whether predicted, by our model, $\hat{y}_i$ is truly describes the empirical data correctly and, consequently, test the forecasting capabilities of our model, it does the empirical data about the real GNP (for American economy) lie within confidence interval, predicted by our model. $y_{n+1} \in \{(\hat{y}_n - st.error * t_{crit});(\hat{y}_n + st.error * t_{crit})\}$.

Let us look at our case:

Table 6

Lower 95%	*Upper 95%*	*Empirical*	*Empirical>Lower 95%*	*Empirical<Higher 95%*
-1,442	3,400	1,584	True	True

So, our empirical for the 1.04 of 1948 data lies between upper and lower boundaries predicted by our model.

We can forecast the future correctly and accurately.

PREDICTING

Judging by the estimated coefficients in the model, the US real GNP is inversely proportional to its level of unemployment.

The previous tests showed that Okun's law has not been useful as a stable relationship, since its parameters have varied considerably over time and over the course of the business cycle. In addition, it has not always been a reliably strong relationship, especially in quarterly data.

CONCLUSION

The previous tests showed that Okun's law has not always been a reliably strong relationship, especially in quarterly data. Nevertheless, the relationship between contemporaneous changes in unemployment and output growth may still be useful to policymakers and economists if they take these shortcomings into consideration. In this way our model is of a good explanatory ability and can be used for general data analysis of consumption functions.