The Value of Theoretical Models for Real-World Market Analysis

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An analysis of whether the original Cournot and Bertrand models of oligopoly competition provide a satisfactory basis for assessing the extent to which real-world markets are effectively competitive.

Introduction

The real-world value of economic models is a topic that is frequently disputed. The traditional Cournot and Bertrand models of oligopoly competition attempt to explain the organization of an economy. Yet how can their conclusions, which are based on an unrealistic world where there is perfect information, symmetric costs, homogenous products and no strategic behavior, have any real-world significance? Many argue that models such as these should be left to textbooks as they have little value in real-world analysis. Models are designed to explain complex observed processes and are therefore subjective approximations of reality. They can never be perfect but using them in economics is imperative even though it has led to the "assume we have a can opener" catchphrase1 to mock economists and other professionals who base their conclusions on unrealistic or unlikely assumptions. On the one hand, if we are to disregard theories because they rely on unrealistic assumptions then economics (and other professions) won't be left with much. Furthermore, what defines an unrealistic assumption from a realistic assumption? On the other hand, the very process of constructing models, and then testing and revising them forces economists and policymakers to tighten their views about how an economy works. After all, Milton Friedman argued that theories with accurate predictions are of great value even though their assumptions may be extremely "unrealistic." This paper analyses the traditional Cournot and Bertrand models in order to assess their direct applicability in market analysis, therefore determining their subsequent value. It is important to note that "value" in itself is a subjective term and is by no means limited to how well a model can be applied to the real world. Whilst the Cournot and Bertrand models may have limited real-world applicability, their value arises from the knowledge they have given economists and regulators about industrial organization and market power in an oligopoly.

An oligopoly market lies in the middle of the extremes of a monopoly and perfect competition. We know that the outcome of perfect competition is price equal to marginal cost and the outcome of a monopoly is marginal revenue equal to marginal cost. The main difference of the oligopoly market is that firms are assumed to take into account interdependencies, which means that their decisions have taken into account what their rivals might do. Moreover we assume that there are many consumers (none large relative to the size of the market), few firms and barriers to entry. At the most basic level (the traditional models of Cournot and Bertrand), decisions are made simultaneously and the product is assumed to be homogeneous. The outcome of an oligopoly market depends upon the strategy, which in turn depends upon the model we are looking at or “the game” that firms are playing when they are competing. Cournot and Bertrand models of oligopoly competition are examples of two

1 The story is as follows: A physicist, chemist, and an economist are stranded on a desert island with no implements and a can of food. The physicist and the chemist each devised an ingenious mechanism for getting the can open; the economist merely said, "Assume we have a can opener"! (Boulding, 1970)

different games firms could be playing as the former focuses on quantity competition and the latter price competition.

Competition authorities recognise the limitations surrounding these models but use their predictions as a basis for theories of harm in competition investigations. Specifically competition authorities rely on oligopoly models to look at the impact of competition between a few firms when there are barriers to entry. For example regulators may map the structure of the market under question to the model assumptions to identify which model is the “best fit”. From this, they consider under which circumstances the model would predict a welfare concern. They are also used in merger analysis, to see whether a proposed merger poses a competition concern. Moreover the traditional Cournot and Bertrand models provide a useful “rule of thumb” and help to set a benchmark for analysis, explaining what can happen in different types of markets.

Once we can establish what happens in the simple Cournot and Bertrand world, we can move on and tweak the assumptions to develop more complex models, which then help to get a closer perspective of reality. Hence, although Cournot and Bertrand themselves may not be very good at explaining the real world, they should not be ignored when it comes to learning about oligopoly competition as they provide a foundation for more in-depth analysis.

The Cournot model

The Cournot model or Cournot duopoly is named after Antoine Augustin Cournot (1801-1877) who was inspired by observing competition in a spring water duopoly (Varian, 2006). It refers to the game whereby there are two firms producing a homogenous product. Each firm faces identical/symmetric costs and there is no co-operation and no entry. These two firms compete simultaneously by choosing output. As they take interdependencies into account, these two firms have reaction functions whereby they make optimal, profit-maximising quantity decisions based on what the other firm is doing. A point on the firm’s reaction function is the best response for that firm, given what the other firm is doing. As each firm has a reaction function, the outcome of this model is a Nash equilibrium where each player in the game has selected the best response (or one of the best responses) with regard to the other players' strategies (Nash, 1950).

Algebraically, we can solve the general case to show the outcome of a Cournot model. If we begin by supposing there are N firms. Price is the same across these N firms and is determined by market demand (Q), where $Q = \sum_{i=1}^{N} q_i$.

$P = a - bQ = a - b\sum_{i=1}^{N} q_i$ where a and b are constants.

We assume symmetric costs for both firms: C(qi) = cqi.

From this we can work out total revenue and individual firm profit by incorporating the residual demand, which is the market demand that is not met by other firms in the industry. In particular, quantities are "strategic substitutes"; if one firm increases output, another firm has lower residual demand for that price.

πi = TR-TC = Pqi− cqi = (a-bΣqii=1…N)qi− cqi = (a-c) qi - bqi 2 - bqiΣ$qji\neq j$

As firms are said to be profit maximising, we can differentiate the equation above and set it equal to zero: $\partial\pi$i∂qi=(a−c)−2bqi−bΣqj − $bqij\neq i$Σ($\partial qj\partial qi$)$j\neq i$

We assume that $\partial qj\partial qi$ = 0 by the zero conjectural variation, which says that if a firm (for example firm A) makes an output decision, we can hold output choices of other firm's (firm B) constant. In other words, firm B won't change it's output choice upon hearing A's output choice.

Therefore : $\partial\pi$i∂qi=(a−c)−2bqi−bΣqj $j\neq i$=0

This rearranges to give: 2bqi= (a−c)−bΣqj $j\neq i$ qi= (a−c)−bΣqj $j\neq i$2b

This is firm i's reaction function. As we can see it depends upon firm j's output choice.

If we assume there are two firms (A and B) we can draw the reaction functions of each and the intersection is where the equilibrium outcome is.

Moreover if we assume symmetry, then each firm produces the same:

q1=q2=….qN

Then we get:

2bqi=(a−c)−b(N−1)qi

qi= (a−c)b(N+1)

This is the outcome decision for individual firms.

As Q is the total output of the economy and we have assumed symmetry, therefore Q=Nqi:
Q=Nqi=N(a−c)b(N+1)

This is aggregate output of the oligopoly market.

To find price we can substitute this into P = a-bQ to get:

P=(a−bQ)= a−[bN(a−c)b(N+1)]= a−[N(a−c)(N+1)]= a+NcN+1

We arrive at the same outcome if we were to simplify P=a-bQ to P = a-Q (i.e. b=1):
P=(a−Q)=a−N(a−c)b(N+1) = aN(b−1)+ab+Ncb(N+1)

If b = 1 this simplifies further to P = $a+NcN+1$

The result of this model is therefore one equilibrium point where firms have no incentive to deviate from; each firm produces the same quantity and charges the same price. It can be shown that the Cournot outcome lies in between the perfect competition and monopoly outcomes. This is useful for competition authorities as it allows them to compare this case to perfect competition and monopoly outcomes and to see the effect on welfare (by calculating the deadweight loss). By changing the number of firms, the effect on welfare and market power can be scrutinised. For instance, further analysis highlights how increasing the number of firms (N), causes the market to perfect competition. This can be useful for competition authorities when they are assessing the effect of, say, a merger.

The main limitation of this model is the naiive conjectural variation assumption which states that $\partial qj\partial qi$ = 0, or in other words that firm i won't change it's output choice upon hearing firm j's output choice.

Realistically, a rival may very well change output upon learning about the output decision of another firm. In addition, in the real world, firms do not face symmetric marginal costs and they do not compete simultaneously. There may be certain markets where this may arise, but in general this isn't the case for most marekts. The Cournot model also ignores the fact that firms might co-operate and collude with each other to reach a more profit maximising outcome. Therefore the Nash equilibrium identified above, won't be true when applied to the real world.

However, out of these concerns, more complex models have been born, such as the Stackelberg game which features sequential moves (a leader and a follower). The outcome of this game is that the leader produces more than the Cournot equilibrium and the follower produces less than the Cournot equilibrium. In addition, we have looked at N firms, an even more basic Cournot model only considers two firms. Therefore there is vast scope for advancing the Cournot model (as mentioned later) to better explain the real world.

The Bertrand model

The Bertrand model, named after Joseph Louis Francois Bertrand (1822-1900), describes a game where firms set prices and it is the consumers that choose quantities at the prices set. Bertrand formulated this model in a review of the Cournot model and found that when firms set prices, the optimal outcome is similar to that of a perfectly competitive world (price equal to marginal cost). Once again this model relies on strict assumptions: homogeneous products; no collusion; simultaneous decision-making and symmetric costs. Similar to the Cournot conjecture, the Bertrand conjecture states that in equilibrium, other firms won't want to change their price choice in response to Firm i's price decision (Edgeworth, 1925).

In order to see the outcome of this game, we assume there are two firms (firm A and firm B). Although we know that they move simultaneously, in order to arrive at a price decision, each firm thinks sequentially. For instance, firm A thinks about making a profit-maximising price decision, taking into account what firm B will respond with. If firm A sets a high price (above marginal cost), firm B will react by charging a slightly lower price (undercutting) and capturing all of the market. As goods are homogeneous, consumers buy from the lowest cost firm. Thus, firm A will then react by undercutting B and this will continue until both arrive at price equal to marginal cost. As they have set the same price, demand is split evenly between them and they each capture half of the market. Therefore this Bertrand-Nash equilibrium is where Pa=Pb=MC and qa=qb=Q/2. The logic is that if A or B charged a price above marginal cost, no one will buy from them and neither would set a price below marginal cost as it would incur a loss (they would rather shut-down and leave the market).

This model is useful because price competition is observed more often than quantity competition. However, this outcome isn't very applicable to a real-world market, as it is based upon unrealistic assumptions that don't arise in most markets. Namely, the zero conjectural variation as seen with the Cournot model, which states that firms won't change their price upon hearing their rival's price. There are many pricing strategies firms may employ that might result in the Bertrand conjecture not holding, such as predatory pricing (where firms deliberately price below their marginal cost, thereby incurring a loss in order to drive rivals out of the market). Furthermore, this model also relies on the assumption that the consumer will always buy from the cheapest firm. Behavioural economists can name a wide variety of reasons as to why this may not be the case; for instance, the quality may be perceived to be greater with a higher price. Another reason could be that consumers are not fully informed and may not necessarily know that there are cheaper options elsewhere.

Do these models hold any value?

Although these models may not be very effective at describing the real world, they do provide a useful benchmark for competition authorities to assess the market with. These traditional oligopoly models, when compared to perfect competition and monopoly cases allow economists to establish a "rule of thumb" which aids competition investigations. For instance, more firms are better (as we get closer to perfect competition) and price competition is a good thing (seen by Bertrand model giving the perfectly competitive outcome of price equal to marginal cost).

The assumptions of perfect information, simultaneous moves and homogenous products limit the applicability of both Cournot and Bertrand models in real-world market analysis. These traditional models have been extended to include asymmetric information as well as sequential moves and product differentiation. Each underlying assumption of the traditional oligopoly models can be changed and the resultant model gets closer to the real world. For instance, Kreps and Scheinkman (1983) brought the two models together to provide a middle ground whereby firms can compete on both quantity and price by choosing a capacity constraint in the first period and a price in the second period. The result is, surprisingly, a Cournot outcome. Hence in this case, it is very important to learn about the Cournot model, as more complex cases may revert back to the Cournot outcome as seen above. Additionally, Cournot and Bertrand can be modified to deal with heterogeneous costs or exogenous product differentiation. The Stackelberg game deals with sequential moves and models also deal with endogenous product differentiation (such as representative consumer models of monopolistic competition and locational models). These more advanced models deal with the limitations of the traditional Cournot and Bertrand models by removing one main assumption and analysing the effect on the outcome. They get closer at explaining the real world, but as they still involve assumptions, their direct applicability is still somewhat limiting. Whether this makes them more valuable and useful than the traditional Cournot and Bertrand models is limited by our definition of “value”. If we are referring to how well these models describe the real world, then sceptics may be justified in their criticisms; perhaps they should be resigned to textbooks. However if value incorporates the usefulness to market analysts, the predictive nature of these models, as well as how they’ve been enhanced to explain more complex economic phenomenon, then their stringent assumptions are their only drawback.

Competition authorities face a trade off when analysing the market, as they want to carry out in-depth analysis in order to arrive at justifiable recommendations but have limited time and resources. Including private information about costs can make the analysis very cumbersome for competition authorities. For the purpose of establishing theories of harm, the traditional Cournot and Bertrand models overcome the trade off competition authorities face with having limited time and resources to analyse the market and arrive at justifiable recommendations.

Although the Cournot model doesn’t necessarily give a realistic view of the world due to its limiting assumptions, it does give competition authorities an idea of whether there might be a competition concern, by allowing them to compare outcomes to a monopoly and perfect competition. Therefore it helps them form theories of harm when beginning the investigation procedure, allowing them to concentrate their analysis on these specific concerns. This makes the investigation process more efficient and less time-consuming as competition authorities are only investigating cases where there is an initial concern.

Conclusion

To conclude, I agree that Cournot and Bertrand offer a biased explanation of the real world, as each are constrained by rigid assumptions which limit their application to reality. However, learning about them is vital as they allow us to develop more complex models, and more importantly they aid competition authorities in assessing whether there are competition concerns that need to be investigated. Although neither traditional models of Cournot or Bertrand have significant direct practical applicability to the majority of real-world markets, these theorems and the outcome they arrive at do have real-world market value as they provide a useful benchmark from which to look at the market. Competition authorities use them to establish theories of harm when carrying out investigations. It is highly unlikely that we’ll ever arrive at a model that will fully explain and help us manipulate our complex world. However the value of these traditional economic models and theories, as Friedman set out in his ‘Methodology of Positive Economics’, lies in the accuracy of their predictions. He maintains that the realism of a theory's assumptions is irrelevant to its predictive value. It does not matter whether the assumptions that firms maximize profits or move simultaneously are realistic. Theories and models should be appraised exclusively in terms of the accuracy of their predictions. The accuracy of each model depends on how well it imitates the industry in question; Bertrand will be better if capacity and output can be easily changed (firms are competing on price) whilst Cournot is generally better if output and capacity are difficult to adjust (and firms are competing on quantity). Thus their value comes from the fact that they form the foundations of our knowledge into how firms behave in oligopoly markets. Moreover it is from the traditional Cournot and Bertrand models that more complex models have been developed; therefore in order to learn and understand the complicated real world, one must be able to understand what happens in a simplified world.

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