The National Differentiation of Products and Ideas

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Abstract

Aggregate returns to scale are a key driver of the gains from trade in both idea-based and variety-based trade models. We argue that aggregate returns to scale depend on the national differentiation of products and ideas. We develop an empirical strategy to identify the parameters governing the degree of national product/idea differentiation. We estimate these parameters across various sectors using micro-level trade data. We find that products and ideas are less differentiated intra-nationally, and that returns to national size are diminishing. We fit a trade model with adjusted scale-effects to bilateral trade data on 101 countries. Our adjusted model explains the observed relationship between population size and real income significantly better than a standard idea or variety-based trade model. Furthermore, our estimates suggest that countries with comparative advantage in instruments (a sector that exhibits stronger returns to aggregate scale) gain relatively more from trade, whereas countries with comparative advantage in textiles (a sector that exhibits weaker returns to aggregate scale) gain relatively less.

1 Introduction

Aggregate returns to scale are at the core of contemporary trade theories and idea-based growth models. The international distribution of the gains from trade is regulated by the degree of *aggregate* returns to scale and its variations across industries. Aggregate scale effects are also a key determinant of the optimal trade tax and have been at the center-stage of many arguments for protectionism. Similarly, scale effects are a prominent feature of innovation-led growth theories. The empirical literature, meanwhile, offers little in terms of estimating parameters that govern these underlying scale effects. Several studies have, however, pointed out that standard trade and growth theories exhibit scale effects that are counterfactually strong (Jones (1999); Rose (2006); Ramondo et al. (2014)).

Introducing domestic trade frictions into standard trade models (Krugman (1980); Eaton and Kortum (2002); Melitz (2003)) usually moderates the overstated scale effects. But even after controlling for direct measures of internal trade frictions, the implied scale effects remain counterfactually strong (Ramondo et al. (2014)).[1] To address this issue, we first argue that aggregate returns to scale depend on the degree to which ideas and products are nationally differentiated. We develop an empirical strategy to identify the parameters governing the degree of national product/idea differentiation. We estimate these parameters across various sectors using micro-level trade data. Our estimates shed light on the relative importance of scale effects across various sectors and provide a foundation for estimating the gains from trade in multi-sector gravity models. Furthermore, we feed our estimates into a standard gravity trade model and demonstrate that this adjustment mitigates the overstated scale effects and revises several key implications of the standard models.

To guide our estimation we develop a discrete choice framework that nests the standard trade models as a special case. Specifically, standard trade models either assume away the intra-national differentiation of products/ideas (Anderson and Van Wincoop (2003); Eaton and Kortum (2002)) or do not distinguish between inter- and intra-national product/idea differentiation (Krugman (1980); Melitz (2003)).[2] Our discrete choice framework, however, delivers an

[1]The other two channels that could also mitigate the overstated scale effects are multinational production and technology diffusion (Ramondo and Rodríguez-Clare (2010)).

[2]Empirical studies that structurally estimates the above models (e.g. Broda and Weinstein (2006), Hummels et al. (2009), Caliendo and Parro (2014), and Soderbery (2015)) generally rely on similar assumptions. The exceptions are Blonigen and Soderbery (2010), Ardelean and Lugovskyy (2010), and Feenstra et al. (2014) who distinguish between the macro and micro elasticities of substitution in a nested CES utility function. Their macro elasticity however, is defined to differentiate between two broader set of varieties – foreign and domestic – whereas we allow for country-specific product differentiation.

import demand function that allows us to separately identify the degrees of inter- and intra-national product/idea differentiation. We use data on the universe of Colombian import transactions to structurally estimate the import demand function across various sectors. The data documents all import transaction into Colombia pertaining to a 10-digit Harmonized System product category in the 2007-2013 period (Importantly, the data identifies all firms exporting to Colombia, allowing us to define a variety as a country-firm-product combination for any given time period). Our estimation utilizes monthly variations in the data and indicates, in contrast to standard normalizations, products and ideas are less differentiated intra-nationally. Our estimates, therefore, imply that returns to aggregate scale are larger than those assumed in perfectly competitive trade models; but smaller than those in monopolistically competitive models. In particular, we find that a 10% increase in population size increases the nominal number of national ideas/varieties proportionally, but increase the effective number of ideas/varieties by less than 8%.

Aggregate economies of scale give rise to aggregate scale effects with important macro-level implications. To study these implications, we construct a gravity trade model that accommodates the adjusted scale effects implied by our micro-level estimation. We fit our adjusted model to bilateral trade data on 101 countries. Our adjusted model correctly predicts the relationship between population size and real per capita income, which is weakly negative. In comparison, a standard Krugman model fitted to the same data exhibits counterfactually strong scale effects, predicting a strong, positive relationship between per capita income and population size.[3] The overstated scale effects in the standard model are so strong that introducing domestic trade frictions only partially mitigates them.

Our estimates also provide a foundation for estimating the gains from trade in multi-sector gravity trade models. The gains from trade in such models are regulated by (1) the trade elasticity and (2) the scale elasticity (which depends on the degree of aggregate returns to scale). The trade elasticity governs the overall size of the gains from trade, and has been estimated extensively in the empirical trade literature (Broda and Weinstein (2006); Simonovska and Waugh (2014); Caliendo and Parro (2014)). The scale elasticity governs the inter-

[3]The Eaton and Kortum (2002) would predicts a similar counter-factual prediction given that the stock of non-rival ideas in any country is proportional to its population size (see Ramondo et al. (2014)).

country distribution of the gains from trade. To the best of our knowledge we are the first to structurally estimate the scale elasticity across a wide range of industries. A straightforward implication of our estimates is that the gains from trade are less pro-small than those predicted by standard trade models. This is because the marginal value of access to new varieties diminishes with the size of the trading partner. Furthermore, our estimates suggest that countries with comparative advantage in instruments (a sector that exhibits stronger returns to aggregate scale) gain relatively more from trade, whereas countries with comparative advantage in textiles (a sector that exhibits weaker returns to aggregate scale) gain relatively less.

The fact that products and ideas are more differentiated intra-nationally, rejects the independence of irrelevant alternatives (IIA), an assumption imposed by many contemporary trade theories. While the IIA assumption has garnered tremendous attention in Industrial Organization literature, the trade literature has only recently tested this assumption against data. Redding and Weinstein (2016) estimate an international demand system that relaxes the IIA assumption by accommodating heterogeneous taste across consumers. Adao et al. (2015) estimate a trade model that (unlike standard CES models) permits varieties from certain countries to be closer substitutes. Our paper contributes to this emerging literature by highlighting another aspect of the trade data that is at odds with the IIA assumption.

At a broader level, this paper contributes to our understanding of aggregate scale economies, which are central to other domains of economics such as idea-based growth theories. Idea-based growth models generally exhibit scale effects that are too strong (Jones (1999)). The national differentiation of products and ideas, Implied by our micro-level estimates, could help reconcile growth theory with data. Our paper also contributes to a vibrant literature that studies the benefits of economic versus political integration (see Alesina et al. (2000) and Alesina and Spolaore (2005)). While a thorough study of this matter is beyond the scope of this paper, our results nevertheless shed new light on the relative advantages of economic integration. Specifically, our calibrated model, implies that full economic integration (free trade across national borders) is 70% more beneficial than political integration (eliminating national borders). The intuition is that ideas and products are more homogeneous within national borders. Preserving national borders, therefore, contributes to the diversity of

Table 1: Summary statistics of the Colombian import data.

Statistic	year						
	2007	2008	2009	2010	2011	2012	2013
F.O.B. value (billion dollars)	30.77	37.26	31.39	38.41	52.00	55.79	56.92
$\frac{\text{C.I.F. value}}{\text{F.O.B. value}}$	1.08	1.07	1.05	1.06	1.05	1.05	1.05
$\frac{\text{C.I.F. + tax value}}{\text{F.O.B. value}}$	1.28	1.21	1.14	1.19	1.15	1.18	1.15
No. of exporting counrties	210	219	213	216	213	221	224
No. of imported varities	483,286	480,363	457,000	509,524	594,918	633,008	649,561

Notes: tax value includes import tariff and value added tax (VAT). The number of varieties corresponds to the number of country-firm-product combination imported by Colombia in a given year.

products and ideas.

2 Patterns of Trade and Real Income

Our primary data source covers daily import transactions from the Colombian Customs Office for the 2007-2013 period.[4] The data include detailed information on each transaction, e.g. Harmonized System 10-digit product category (HS10), importing and exporting firms, f.o.b. (free on board) and c.i.f. (customs, insurance, and freight) values of shipments in US dollars, quantity, unit of measurement (of quantity), freight in US dollars, insurance in US dollars, country of origin, and weight. Table 1 shows summary statistics of these data.

We also use aggregate statistics from other sources: (i) aggregate bilateral merchandise trade flows from the U.N. COMTRADE database (Comtrade (2010)); (ii) national account data from the World Bank database (World-Bank (2012)); (iii) and data on bilateral distance, common official language, and borders from Mayer and Zignago (2011). Below, we present three empirical facts about to the relationship between population size, the number of exported varieties, export values, and real per-capita income.

Pattern 1 *With the rise of international trade, the relation between real income per-capita and population size has transformed from weakly positive in the 1960s to nega-*

[4]The data is obtained from Datamyne, a company that specializes in documenting import and export transactions in Americas. For more detail please see www.datamyne.com.

tive in the late 2000s.

Figure 1 illustrates the above pattern. Figure The bottom panel plots the relationship between the (expenditure side) real income per-capita and population size for years 2008 and 1960. The relationship is weakly positive in 1960, but negative in 2008. The decline of the *size-income* relationship coincides with the rise of international trade (figure 1, top panel). Overall, these trends align with previous studies that find no systematic relation between population size and national prosperity (e.g. Rose (2006)). This observations, however, cannot be easily reconciled with standard gravity trade models. Typically, trade models predict that international trade should weaken the effect of population size on the standard of living. However, a standard trade model fitted to trade data in 2008 will nevertheless predict a positive relationship between population size and real per capita income (see section 5). In other words, the national scale-effects in standard trade models are too strong to be eliminated by the observed levels of economic integration.

Pattern 2.A *The nominal number of exported varieties increase proportionally with population size...*

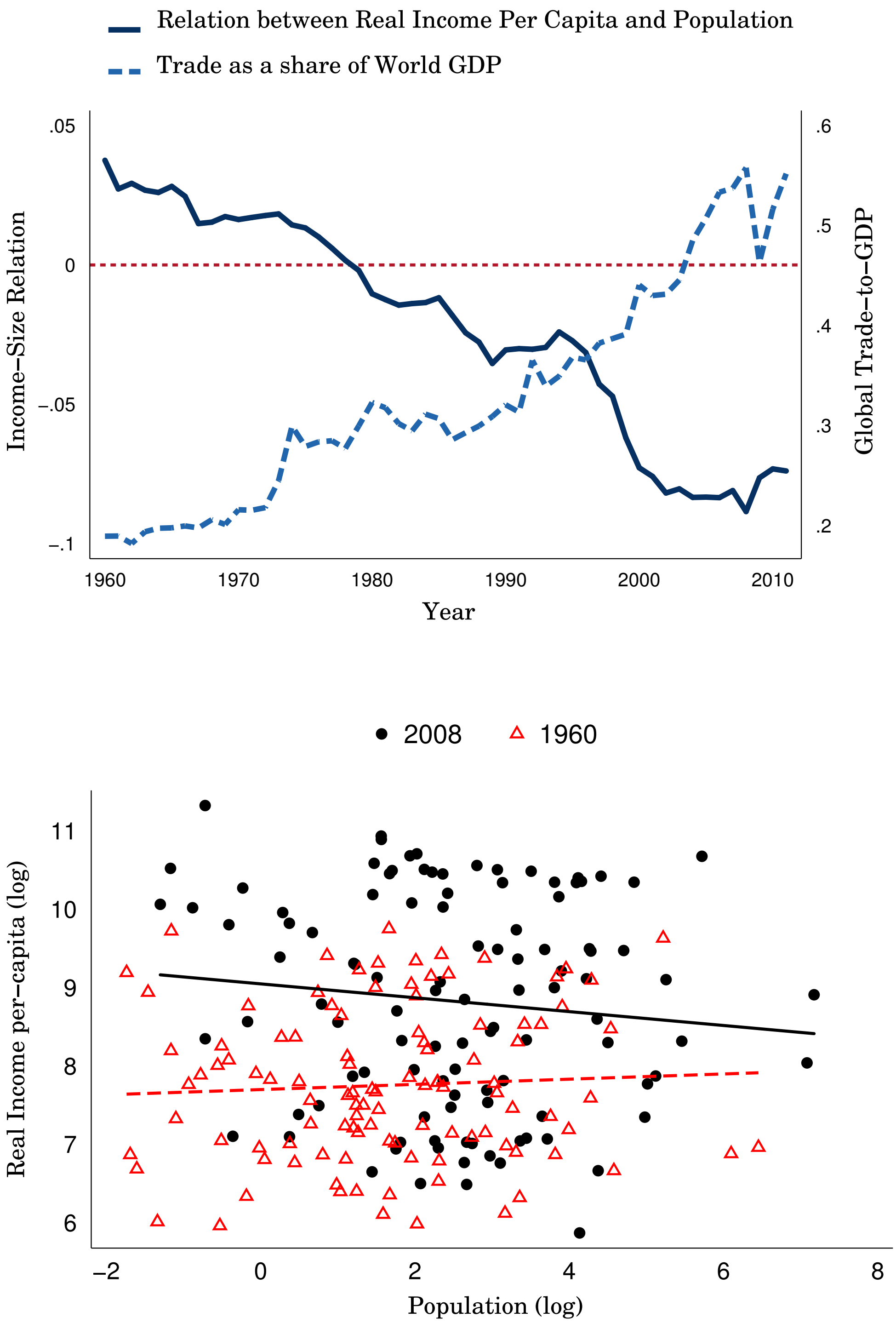
We establish this fact using our micro-level import data from Colombia. We define a variety as a country-firm-product combination imported by Colombia in a given year. For example, in 2007, a product with an HS10 code 8428101000[5] was exported to Colombia by firm *"MITSUBISHICO"* from two countries of origin, Japan and Thailand. We treat these as two distinct varieties: one exported to Colombia by Japan and one exported by Thailand. Based on this definition, we calculate the total number of *firm×product-specific* varieties imported (by Colombia) from country i in year t, N_{it}. We then run the following regression:

$$\ln N_{it} = \beta_t + \beta_P \ln \text{Population}_{it} + \beta_Y \text{GDP per capita}_{it} + \beta_D \text{Dist}_i + \beta_B \text{Border}_i + \varepsilon_{it} \quad (1)$$

where β_t denotes a year dummy. The results (displayed in table 2) suggest that the number of exported varieties increase proportionally with population size, which supports the prediction of a standard variety-based trade model

[5]Which is described as *"Ascensores sin cabina ni contrapeso."*

Figure 1: The rise of international trade has weakened the relationship between real per-capita income and population size.



Notes: The top panel suggests that post 1980, larger countries (on average) have a lower real income per-capita. The bottom panel plots the size-income relationship for years 1960 and 2006.

Table 2: *The number of exported varieties vs. National characteristics*
Dependent variable: Number of exported varieties (log)

	Regressor (log)					
Sample	Population	GDP per capita	Distance	Border	R^2	Obs.
All Products	1.01***	1.15***	-0.56***	1.62***	0.68	1019
	(0.03)	(0.03)	(0.07)	(0.33)		
Manufacturing	1.01***	1.12***	-0.44***	1.86***	0.66	983
	(0.03)	(0.04)	(0.07)	(0.31)		

(e.g. Krugman (1980) and Melitz (2003)). Further, this fact aligns well with the assertion in Eaton and Kortum (2001) that the stock of (frontier) ideas in a country is proportional to population size. Existing theories, however, usually impose an additional assumption: export values increase proportionally with the number of exported variates (or the stock of frontier ideas). Put differently, the effective number of varieties is assumed to be equal to the nominal number of varieties. The next pattern we document does not align with this standard assumption.

Pattern 2.B *... But export shares increase less-than-proportionally with the number of exported varieties.*

To establish the above pattern we estimate the elasticity of X_{it} (total exports of country i in year t) with respect to N_{it} (the total number of varieties exported by country i in year t):

$$\ln X_{it} = \phi_t + \phi_N \ln N_{it} + \varepsilon_{it}$$

Since the number of exported varieties to Colombia is endogenous, we instrument it with population, GDP per capita, border, and bilateral distance (regression 1, therefore, describes the first-stage estimation). The two-stage least square estimates are displayed in table 3, suggesting that export values increase less-than-proportionally with the number of exported varieties. That is to say, while the *nominal* number of exported varieties increases proportionally with population size, the *effective* number of varieties increases less-than-proportionally. This result stands in sharp contrast to the standard *"love of vari-*

Table 3: *Export value vs. The number of exported varieties.*

Dependent: Total Export Value to Colombia (log)			
Sample	Number of Exported Varieties (log)	R^2	Obs.
All Products	0.69*** (0.03)	0.79	1019
Manufacturing	0.74*** (0.03)	0.83	983

ety" specification in Krugman (1980), or the assertion the number of efficiency-enhancing ideas is proportional to the actual stock ideas in a country (e.g. Eaton and Kortum (2001) and Ramondo et al. (2014)). The next section argues the these *proportionality* assumptions are analogous to assuming independence of irrelevant alternatives (IIA). Pattern 2.B, therefore, simply suggests that the IIA assumption might be too restrictive.

Note that our analysis of the number of exported varieties and country size is closely related to the Hummels and Klenow (2005) in finding that the number of exported varieties increases less than proportionately with the country size. While we use different measures of the numbers of variety and different dataset,[6] our main result coincides with theirs: the effective number of varieties increases less than proportionately with the country size. This is not surprising, since their measure of the number of varieties includes export value, as does our measure of the effective number of varieties. The novelty of our paper is that we distinguish between the effective and the nominal numbers of varieties and explain different patterns of variety growth with the country size (the nominal number of varieties increases proportionately with the country size).

[6]Our country-firm-product measure of variety is more disaggregated than the country-product based measure of variety in Hummels and Klenow (2005). We also consider two measures of the numbers of varieties – nominal and effective – where the nominal one does not depend on the export value, while the effective does.

3 Theory

This section presents a discrete choice framework that nests both the variety-based gravity models (Krugman (1980); Chaney (2008)) and the idea-based gravity models (Eaton and Kortum (2001, 2002)). Guided by this framework we drive a structural import demand function, which we estimate using our micro-level trade data.

The Environment. there are N countries, with C denoting the set of countries. There are S product categories indexed by s. The utility of consumer n in country $i \in C$ is a Cobb-Douglas aggregator across the S categories

$$U(n) = \prod_{s=1}^{S} U^s(n)^{\alpha_s}$$

where $U^s(n)$ denotes the sub-utility corresponding to product s, and α_s is the share of income spent on product s, with $\sum_{s=1}^{S} \alpha_s = 1$. Within-product consumption shares are governed by a discrete choice structure, which has two interpretations: (i) a cost differentiation interpretation corresponding to idea-based gravity models or (ii) a taste differentiation interpretation corresponding to variety-based gravity models.

3.1 Variety-based model

Variety-based trade models, exemplified by Krugman (1980), are structured around horizontal differentiation (i.e. taste-heterogeneity). Specifically, there is a mass L_i of individuals in country i. Each individual is endowed with one unit of labor, which is the the only factor of production. Individuals are heterogeneous in their taste for firm-specific varieties. Each individual chooses one unique variety in category s, and spends α_s share of their income exclusively on that variety. The utility individual n in country i derives from variety ω in category s is given by:

$$U^s_\omega(n) = \ln z^s_\omega . q^s_\omega(n) + \epsilon^s_\omega(n)$$

where $q^n_\omega(n)$ denotes the quantity consumer n can purchase of variety ω; z^s_ω denotes the quality of variety ω, and $\epsilon^z_\omega(n)$ denotes the taste of consumer n for that variety. Individual n endowed with income $y(n)$, therefore, solves the following problem:

$$\begin{aligned} U^s_n \;=\; & \max_\omega \;\; \ln z^s_\omega . q^s_\omega(n) + \epsilon^s_\omega(n) \\ \text{s.t.} \quad & p^s_{\omega ji} q^s_\omega(n) = \alpha_s y(n) \end{aligned}$$

where $p^s_{\omega ji}$ denotes the price of variety ω, produced in country j and sold in country i. Taste, $\epsilon^s_\omega(n)$, is drawn *i.i.d.* from the following General Extreme Value (GEV) distribution:

$$F^s(\boldsymbol{\epsilon}) = \exp\left(-\sum_{j=1}^{N} \left(\sum_{\omega \in \Omega^s_{ji}} e^{-\theta_1 \epsilon_\omega} \right)^{\frac{\theta_2}{\theta_1}} \right)$$

Following Manski et al. (1981), the share of consumers who choose variety ω will given by (see appendix A):

$$\lambda^s_{\omega ji} = \lambda^s_{\omega|ji} \times \lambda^s_{ji} \tag{2}$$

where $\lambda^s_{\omega|ji}$ is the share of consumers who chose variety ω conditional on buying from country j:

$$\lambda^s_{\omega|ji} = \frac{\left[p^s_{\omega ji}/z^s_\omega\right]^{-\theta_1}}{\sum_{\omega' \in \Omega^s_{ji}} \left[p^z_{\omega' ji}/z^s_{\omega'}\right]^{-\theta_1}} = \left(\frac{p^s_{\omega ji}/z^s_\omega}{P^s_{ji}} \right)^{-\theta_1} \tag{3}$$

where $P^s_{ji} \equiv \left[\sum_{\omega' \in \Omega^s_{ji}} \left(p^s_{\omega' ji}/z^s_{\omega'} \right)^{-\theta_1} \right]^{-\frac{1}{\theta_1}}$ denotes the price index of varieties exported from country j to i in category s. λ^s_{ji} is the share of consumers in country i who choose varieties originating from country j:

$$\lambda^s_{ji} = \frac{\left(\sum_{\omega \in \Omega^s_{ji}} \left[p^s_{\omega ji}/z^s_\omega\right]^{-\theta_1} \right)^{\frac{\theta_2}{\theta_1}}}{\sum_{k=1}^{N} \left(\sum_{\omega' \in \Omega^s_{ki}} \left[p^s_{\omega' ki}/z^s_{\omega'})\right]^{-\theta_1} \right)^{\frac{\theta_2}{\theta_1}}} = \left(\frac{P^s_{ji}}{P^s_i} \right)^{-\theta_2} \tag{4}$$

where $P_i^s \equiv \left[\sum_{k=1}^{N}\left(P_{ki}^s\right)^{-\theta_2}\right]^{-\frac{1}{\theta_2}}$ denotes the price index of product k in country i (which is aggregated across all source-specific price indexes). The total expenditure (in US dollars) on variety ω in country i will, therefore, be:

$$X_{\omega ji}^s = p_{\omega ji}^s \left(\int_n \frac{\alpha_s y(n)}{p_{\omega ji}^s} \lambda_{\omega ji}^s dn\right) = \lambda_{\omega ji}^s X_i^s \tag{5}$$

where $X_i^s \equiv \alpha_s \int_n y(n)dn$ denotes the total expenditure on product s in country i. The above equation, therefore, implies that the market share of variety ω (in dollar value) is also equal to $\lambda_{\omega ji}^s$.

Krugman (1980). The variety-based trade model developed by Krugman (1980), is a single-sector ($S = 1$) case of the above framework, where firms from a given country j are assumed to be symmetric and monopolistically competitive. Labor is the only factor of production and the unit cost of producing a given variety ω in country j and selling it to country i is $\tau_{ji}w_j$; where τ_{ji} denotes the iceberg trade cost and w_j denotes the labor cost. All firms from country j, therefore, charge the following price and offer the same quality:

$$\begin{cases} p_{\omega ji} = \left(\frac{\theta_1+1}{\theta_1}\right)\tau_{ji}w_j \\ z_{\omega ji} = z_j \end{cases}$$

The number of firms in country i is disciplined by the *free entry condition*, which implies that the number of varieties/firms is proportional to population size:

$$M_i = \frac{L_i}{(\theta_1 + 1) f^e}$$

Finally, given that labor is the only factor of production, it follows that $X_i = w_i L_i$. Plugging the above equations into equation 5, results in the following gravity equation:

$$X_{ji} = \frac{L_j^{\frac{\theta_2}{\theta_1}} \left(\tau_{ji}w_j/z_j\right)^{-\theta_2}}{\sum_{k=1}^{N} L_k^{\frac{\theta_2}{\theta_1}} \left(\tau_{ki}w_k/z_j\right)^{-\theta_2}} w_i L_i$$

Krugman (1980) assumes that $\frac{\theta_2}{\theta_1} = 1$, i.e. there is no scope for national product differentiation. Alternatively, when $\frac{\theta_2}{\theta_1} \to \infty$ the model reduces to the perfectly competitive Armington gravity model, analyzed in Anderson (1979).[7]

3.2 Idea-based model

Idea-based trade models, exemplified by Eaton and Kortum (2002), accommodate price heterogeneity, which is driven by cost differentiation. Such models feature a continuum of goods rather than a continuum of consumers. In particular, there are a continuum of goods (indexed by n) in category s. The utility of the representative consumer in country i within category s is described by

$$U_i^s = \int_0^1 \ln \tilde{q}_i^s(n) dn.$$

Country j is comprised of various entities/individuals indexed by ω. Each entity can supply a given good (n) to market i at the following effective price:

$$\tilde{p}^s_{\omega ji} = \frac{p^s_{\omega ji}}{z^s_\omega}$$

where $p^s_{\omega ji}$ is the deterministic comportment common to all goods, and z_ω is an idiosyncratic component driven independently for each good from a two-tier Fréchet distribution:

$$F^s(z) = \exp\left[-\sum_k \left(\sum_{\omega\in\Omega_k} \tilde{T}_i^s z_\omega^{-\theta_1}\right)^{\frac{\theta_2}{\theta_1}}\right]$$

where $\tilde{T}_i^s \equiv \left(T_i^s\right)^{\theta_1}$. The Fisher–Tippett–Gnedenko theorem states that if ideas are drawn from a (normalized) distribution, in the limit the distribution of the best draw takes the form an general extreme value (GEV) distribution, which nests the above Fréchet distribution as a special case. Kortum (1997) further

[7] Similarly, if we introduce firm heterogeneity into the Krugman model (à la Melitz (2003)) and assume that firm-level productivity has a Pareto distribution with shape parameter γ the gravity equation becomes

$$X_{ji} = \frac{L_j\left(\tau_{ji}w_j\right)^{-\gamma} f_{ji}^{\frac{\theta-\gamma}{\theta}}}{\sum_{k=1}^N L_k\left(\tau_{ki}w_k\right)^{-\gamma} f_{ki}^{\frac{\theta-\gamma}{\theta}}} w_i L_i \,,$$

where $\theta \equiv \theta_1 = \theta_2$ and f_{ji} denotes the fixed cost of exporting from country j to i.

shows that T_i^s (in the above distribution) represents the stock of ideas in country i, sector s.

The representative consumer in county i is endowed with income X_i. For each good n, the representative consumer in i maximizes the effective quantity purchased ($\tilde{q}_i^s = \frac{X_i}{\tilde{p}_{ji}^s}$) by sourcing from the cheapest supplier. This, corresponds to solving the following the discrete choice problem:

$$\tilde{p}_i^s(n) = \min_{\omega}\left\{\tilde{p}_{\omega ji}^s(n)\right\}$$

The theorem of general extreme value implies that the share of goods purchased from source ω in country j is equal to (see appendix A)

$$\lambda_{\omega ji}^s = \left(\frac{p_{\omega ji}^s / T_i^s}{P_{ji}^s}\right)^{-\theta_1}\left(\frac{P_{ji}^s}{P_i^s}\right)^{-\theta_2} \tag{6}$$

The total exports of firm ω (located in country j) to country i will be:

$$X_{\omega ji}^s = p_{\omega ji}^s q_{\omega ji}^s = p_{\omega ji}^s \lambda_{\omega ji}^s \frac{X_i}{p_{\omega ji}^s} = \lambda_{\omega ji}^s X_i$$

Eaton and Kortum (2002). Consider a one sector case ($S = 1$) of the above model where each country is populated with L_i symmetric individuals who draws ideas and produce goods — labor is the only factor of production. The common price term $p_{\omega ji}$ depends on the iceberg trade cost and the labor cost: $p_{\omega ji} = \tau_{ji} w_j$. Equation 6 implies that export flows from country j to i are given by the following gravity equation:

$$X_{ji} = \frac{\tilde{T}_j L_j^{\frac{\theta_2}{\theta_1}} \left(\tau_{ji} w_j\right)^{-\theta_2}}{\sum_{k=1}^N \tilde{T}_k L_k^{\frac{\theta_2}{\theta_1}} \left(\tau_{ki} w_k\right)^{-\theta_2}} X_i$$

where $\tilde{T}_i \equiv T_i^{\theta_1}$, and $X_i = w_i L_i$ is total income in country i. Eaton and Kortum (2002) implicitly assume that the productivity draws are perfectly correlated within a country, i.e. $\theta_1 \to \infty$. Eaton and Kortum (2001) and Ramondo et al. (2014), by contrast, assume the same degree of inter- versus intra-national technology differentiation: $\theta_1 = \theta_2$.

4 Estimation

This section estimates the structural parameters $\frac{\theta_1}{\theta_2}$ and θ_2 using transaction level Colombian import data. To this end, we use the general discrete choice framework presented above, which accommodates both price and taste heterogeneity. Specifically, we directly observe data on import prices, and use our discrete choice framework to impose structure on the unobserved taste (or quality) heterogeneity. Equations 2, 3, and 4 imply that the sales of firm ω located in country j to market i are given by:

$$x^s_{\omega ji} = \left(\frac{p^s_{\omega ji}/z^s_{\omega ji}}{P^s_{ji}}\right)^{-\theta_1} \left(\frac{P^s_{ji}}{P^s_i}\right)^{-\theta_2} X^s_i$$

Rearranging the above equation and taking logs, would give us the following log-linear import demand unction:

$$\ln x^s_{\omega ji} = -\theta_2 \ln p^s_{\omega ji} + \left(1 - \frac{\theta_2}{\theta_1}\right) \lambda^s_{\omega|ji} + \Phi^s_i + \epsilon^s_{\omega ji} \qquad (7)$$

where $\Phi^s_i \equiv \ln X^s_i + \theta_2 \ln P^s_i$, and $\epsilon^s_{\omega ji} \equiv \theta_1 \ln z^s_{\omega ji}$. Notice that in the above equation θ_2 corresponds to the international trade elasticity, which is a key determinant of the gains from trade. $1 - \frac{\theta_2}{\theta_1}$ captures the the degree of national product/idea differentiation and is identified as follows. If conditional on having the same price all variates (from all countries) absorb the same market share, goods are not nationally differentiated. However, goods are nationally differentiated if conditional on price, varieties from crowded sources (low $\lambda^s_{\omega|ji,t}$) absorb less market share. That is, conditional on price, a Chinese firm will absorb less market share than a Taiwanese firms, because there are various other Chinese varieties to choose from and Chinese varieties are perceived to be more similar to one another.

We estimate equation 7 using a first difference estimator. Specifically, our estimating equation is[8]

$$\Delta \ln x^s_{\omega ji,t} = -\theta_2 \Delta \ln p^s_{\omega ji,t} + \left(1 - \frac{\theta_2}{\theta_1}\right) \Delta\lambda^s_{\omega|ji,t} + \mu_t + \varepsilon^s_{\omega jit}$$

[8]The implicit assumption is that the product-specific and time-specific components of Φ^s_i are additively separable: $\Phi^s_{i,t} = \phi^s_i + \phi_{i,t}$. Therefore, $\Delta\Phi^s_{it} = \phi_{i,t} - \phi_{i,t-1} \equiv \mu_t$.

Price and conditional market share are both endogenous to variation in the demand shifter. To handle endogeneity, we construct firm-specific instruments. To this end, we utilize that fact that our data reports multiple import transactions for a given variety in a given year. We construct two instruments that exploit this variation: trade-weighted exchange rate and trade-weighted import tax. For example, the trade-weighted exchange rate is constructed as follows:

$$e^s_{\omega ji,t} = \frac{\sum_{m \in t} e^s_{\omega ji,m} x^s_{\omega ji,m}}{\sum_{m \in t} e^s_{\omega ji,m}}$$

where m denotes a month in year t and e denotes the exchange rate in that month. Effectively, firms face different exchange rates depending on which months of the year they export in. The trade-weighted import tax is constructed in a similar fashion, but provides more variation given that taxes change both over time and across firms. We also construct an additional instrument for price, which is the average price charged by firm ω in parallel years in parallel HS10 product categories. The estimation results reported in the next section are produced with trade-weighted import tax as the instrument. However, all the estimates remain qualitatively the same when we instead use trade-weighted exchange rates and parallel prices as instruments. For the conditional market share, $\lambda^s_{\omega|ji,t}$, we adopt instruments that are similar to those suggested by Khandelwal (2010). In particular, we instrument for conditional market share with (i) the number of firms exporting from country j in product category s, (ii) the total number of product categories firm ω exports in year t.

Results. The estimation results (displayed in table 4) imply $\frac{\theta_2}{\theta_1} \approx 0.8$, rejecting the independence of irrelevant alternatives (IIA). That is, products and ideas are relatively more similar intra-nationally. The degree of national product/idea differentiation is, however, not as extreme as assumed in Anderson (1979) (the Armington model) or Eaton and Kortum (2002). Additionally, we find that products and ideas are more nationally differentiated in the manufacturing sector. Our findings are qualitatively the same when we run the estimation separately for various 2-digit product categorizes. The result reported in table 5 suggest that all categories, except for footwear/headgear and leather, exhibit a statistically significant degree of national differentiation. The degree of national differentiation is estimated to be the highest for chemicals and textiles. Also, note that the estimated trade elasticities (θ_2) are slightly lower than those

Table 4: Import Demand Estimation

	All		Manufacturing	
Variable	IV	OLS	IV	OLS
log Price, $-\theta_2$	-2.145***	0.389***	-2.517***	0.416***
	(0.058)	(0.001)	(0.015)	(0.002)
Conditional market share, $1-\frac{\theta_2}{\theta_1}$	0.211***	0.397***	0.224***	0.396***
	(0.012)	(0.002)	(0.015)	(0.002)
Observations	1,332,873	1,332,873	1,129,891	1,129,891
R-squared	...	0.295	...	0.295

Notes: The estimation is conducted with year dummies, which are not reported.

estimated in Caliendo and Parro (2014) and Simonovska and Waugh (2014). This is expected given that (i) we are exploiting within-HS10 product, across-firm variations rather than within-industry, across-country variations, and (ii) we are employing a distinct identification strategy.

4.1 Discussion

Our estimated degree of national product/idea differentiation ($1-\frac{\theta_2}{\theta_1}$) govern the level of scale-effects in a given sector. In particular, a higher $\frac{\theta_2}{\theta_1}$ implies stronger scale effects. To put our results in perspective, note that standard trade models either assume away scale effects by setting $\frac{\theta_2}{\theta_1}=0$ (e.g. Eaton and Kortum (2002); Anderson (1979)) or normalize the scale effects by setting $\frac{\theta_2}{\theta_1}=1$ (e.g. Krugman (1980); Eaton and Kortum (2001); Melitz (2003)) — see table 6. Our estimates suggest that scale-effects are quantitatively significant, but weaker than generally assumed: $0<\frac{\theta_2}{\theta_1}<1$. As we show in the following section, Our estimates imply that returns to national size are diminishing (i.e. export shares and efficiency levels increase with *national* population size at a diminishing rate). Further, we illustrate that a trade model with adjusted scale-effects explains the observed relationship between population size and real income significantly better than standard trade models with non-adjusted scale effects.

It is worth mentioning that our estimates lend themselves to alternative and possibly broader interpretations of scale effects. For example, our estimates

Table 5: *Import Demand Estimation by Sector*

Sector	HS2 codes	Estimated Parameter θ_2	$1-\frac{\theta_1}{\theta_2}$	Observations
Chemicals	28-38	0.378***	0.327***	60,782
		(0.108)	(0.028)	
Plastics/Rubbers	39-40	1.162***	0.200***	120,733
		(0.106)	(0.034)	
Leather	41-43	3.046***	0.050	14,839
		(0.488)	(0.129)	
Wood Products	44-49	1.203***	0.167***	40,491
		(0.328)	(0.052)	
Textiles	50-63	1.224***	0.333***	101,991
		(0.104)	(0.018)	
Footwear/Headgear	64-67	5.687***	0.157	18,821
		(0.908)	(0.123)	
Stone/Glass	68-71	1.030***	0.225***	32,801
		(0.171)	(0.045)	
Metals	72-83	1.954***	0.155***	160,299
		(0.135)	(0.034)	
Machinery/Electrical	84-85	3.178***	0.229***	471,805
		(0.150)	(0.031)	
Transportation	86-89	1.544***	0.252***	63,760
		(0.223)	(0.060)	
Instruments	90-93	3.152***	0.146**	91,044
		(0.435)	(0.074)	
Miscellaneous	94-96	3.375***	0.233**	62,905
		(0.302)	(0.093)	

Table 6

Idea-based gravity models		Variety-based gravity models	
Eaton and Kortum (2002)	Eaton and Kortum (2001)	Armington	Krugman (1980)/Melitz (2003)
$\frac{\theta_2}{\theta_1} = 0$	$\frac{\theta_2}{\theta_1} = 1$	$\frac{\theta_2}{\theta_1} = 0$	$\frac{\theta_2}{\theta_1} = 1$

could be alternatively interpreted as a sign of diminishing Marshallian externalities, which are highlighted in Kucheryavyy et al. (2015). Such externalities affect the number of varieties and the cost and quality of production within countries. Our empirical strategy controls for the number of varieties and prices, and finds that varieties from crowded sources are discounted by consumers. These findings suggest that benefits from local spillovers could be diminishing given that firms are more similar to their local rivals in terms of the ideas/products they develop.

The above estimates shed light on the size and distribution of the gains from trade. Theoretically, it is well-understood that the gains from trade depend on inter-industry variations in scale-effects (see Costinot and Rodríguez-clare (2014)). In particular, countries that enjoy comparative advantage in industries with strong scale effects would enjoy relatively larger gains from trade. While these implications are theoretically well-understood, they have not been explored empirically, due to lack of empirical estimates on the level of scale effects. Our empirical analysis provide such estimates for a wide range of industries. We find that scale effects indeed vary considerably across industries (they are stronger in leather and footwear and weaker in chemicals and textiles). Our estimates, therefore, offer a foundation to quantify the gains from trade in the presence industry-specific scale effects.

5 General Equilibrium Analysis

Our micro-level estimation rejects the independence of irrelevant alternatives, and confirms the prevalence of national product/idea differentiation. Further, our estimates suggest that scale-effects are present but weaker than those assumed in standard idea or variety-based trade models. This section evaluates the aggregate implications of our estimated scale-effects. We calibrate a general equilibrium gravity trade model with *adjusted* scale effects to bilateral trade data on 101 countries. We compare the explanatory power and the implications of our benchmark model to a that of a standard idea or scale-based trade model with *non-adjusted* scale effects.

Calibration procedure. We calibrate a one-sector version of our model to match bilateral trade shares for 101 countries.[9] The calibration is performed along the following steps:

i. Trade shares are given by

$$\lambda_{ji} = \frac{T_j L_j^{\frac{\theta_2}{\theta_1}} \left(\tau_{ji} w_j\right)^{-\theta_2}}{\sum_{k=1}^{N} T_k L_k^{\frac{\theta_2}{\theta_1}} \left(\tau_{ki} w_k\right)^{-\theta_2}}$$

ii. We data on have L_j (population size in country j), and impose the following parametrization of T_j and τ_{ji}:

$$\begin{cases} \tau_{ji} = \beta_1 \left(\text{DIST}_{ji}\right)^{\beta_2} \\ T_j = \left(H_j\right)^{\beta_3} \end{cases}$$

where DIST_{ji} is distance between countries j and i , and H_j denotes the stock of human capital in country j. Importantly, H_j is not statistically correlated with population size, L_i. Thus, our calibration is distinct from Alvarez and Lucas (2007) and Waugh (2010) who avoid strong scale effects by allowing technology levels decrease rapidly with population size.

iii. For any given vector of parameters $\boldsymbol{\Theta} = \left(\beta_1, \beta_2, \beta_3, \frac{\theta_2}{\theta_1}\right)$ we can solve for a unique vector of wages $\left\{w_j\right\}_{j=1}^{N}$ that satisfies the balanced trade condition. Given the equilibrium wages, we can solve for the equilibrium trade shares.

iv. We search for a vector of parameters $\boldsymbol{\Theta}$ that minimize the distance between predicted trade shares and factual trade shares. Note that while we choose $\frac{\theta_2}{\theta_1}$ to fit aggregate trade data, we could alternatively calibrate

[9] We restrict our attention to 101 countries for which we have data on the stock of human capital in 2000.

it to the values implied by our micro-level estimation. Both approaches deliver they same outcomes, qualitatively.

We compare our calibrated model to a baseline model, which imposes normalized scale effects: $\frac{\theta_2}{\theta_1} = 1$. The baseline model is, therefore, isomorphic to the idea-based gravity model in Eaton and Kortum (2001) and the variety-based gravity model in Krugman (1980).

5.1 Quantifying the Role of Adjusted Scale Effects

We use our calibrated model to quantify the aggregate implications of adjusted scale effects. Our quantitative analysis delivers three principal results, which we discuss below.

National size and welfare Unadjusted effects in standard idea or scale-based trade models imply non-diminishing returns to national size. As a result, standard gravity trade models predict a counter-factually strong and positive relationship between population size and real per-capita income. Specifically, in a standard gravity model, real income per worker in country i (W_i) depends on the technology level (T_i), population size (L_i), and domestic expenditure share (λ_{ii}):

$$W_i = \gamma \times (T_i \cdot L_i)^{\frac{1}{\theta_2}} \times \lambda_{ii}^{-\frac{1}{\theta_2}}$$

As pointed out by Ramondo et al. (2014), small countries import a higher share of their income (i.e. have a lower λ_{ii}) which partially mitigates their size disadvantage. However, after controlling for the observed levels of trade, unadjusted scale effects are strong enough to imply a strong, positive relationship between national size and real income per worker. Figure 2 (bottom panel) displays the positive size-income relationship predicted by the standard gravity model calibrated to aggregate trade data.

In our benchmark model scale effects are adjusted to align with our micro-level estimates. Real income per worker in our adjusted model is given by

$$W_i = \gamma \times T_i^{\frac{1}{\theta_2}} \times L_i^{\frac{1}{\theta_1}} \times \lambda_{ii}^{-\frac{1}{\theta_2}}$$

Our micro-level estimates (and our calibration) imply that $\theta_1 > \theta_2$, which corresponds to weaker scale effects and, therefore, a weaker relationship between national size and real income. As illustrated in figure 2, our benchmark model with adjusted scale effects could reproduce the negative relationship between national size and real per-capita income, which is implied by data. To give an example, consider a small, rich country like Sweden. Our benchmark model implies a real per-capita income for Sweden that is 60% of the United States's, much closer to the data (76%) than a standard gravity model with non-adjusted scale effects (30%). Notice that in our calibrated model scale effects are dampened by national product/idea differentiation. Alternatively, domestic frictions could also dampen scale effects, a force highlighted extensively in Ramondo et al. (2014).

The gains from trade and national size Scale effects are a key driver of the gains from trade. If scale effects vary across industries (as our micro-level estimation suggest) then countries that have comparative advantage in industries with stronger scale effects would benefit relatively more from trade (see Costinot and Rodríguez-clare (2014)). Our micro-level estimates provide a basis for quantifying such effects. However, an analysis of that magnitude is beyond the scope of our calibration exercise. Nevertheless, our calibrated model sheds new light on the link between the gains from economic integration and country size. Specifically, our benchmark model (with adjusted scale effects) predicts that the gains from economic integration (moving from the factual trade equilibrium to free trade) are both smaller and less pro-small. Intuitively, economic integration exposes countries to new ideas/products developed by foreign populations. In a standard idea/variety-based trade model free trade with partner i exposes one to L_i effectively new ideas/products. As a result small countries opening to trade with large partners expose themselves to relatively more new ideas and gain relatively more from trade. In our benchmark model with adjusted scale effects, free trade with partner i exposes one to $L_i^{\theta_2/\theta_1}$ effectively new ideas/products (where $\frac{\theta_2}{\theta_1} < 1$). Hence, the benefits of trade with larger partners are relatively smaller than the standard (non-adjusted). Figure 3 shows these difference, which are quantified using our calibrated model.

Economic integration versus political integration Our calibrated model sheds new light on the benefits of political disintegration and the optimal size of na-

Figure 2: Real wage: our model (LL) versus the standard scale/idea-based model (Krugman) versus the data.

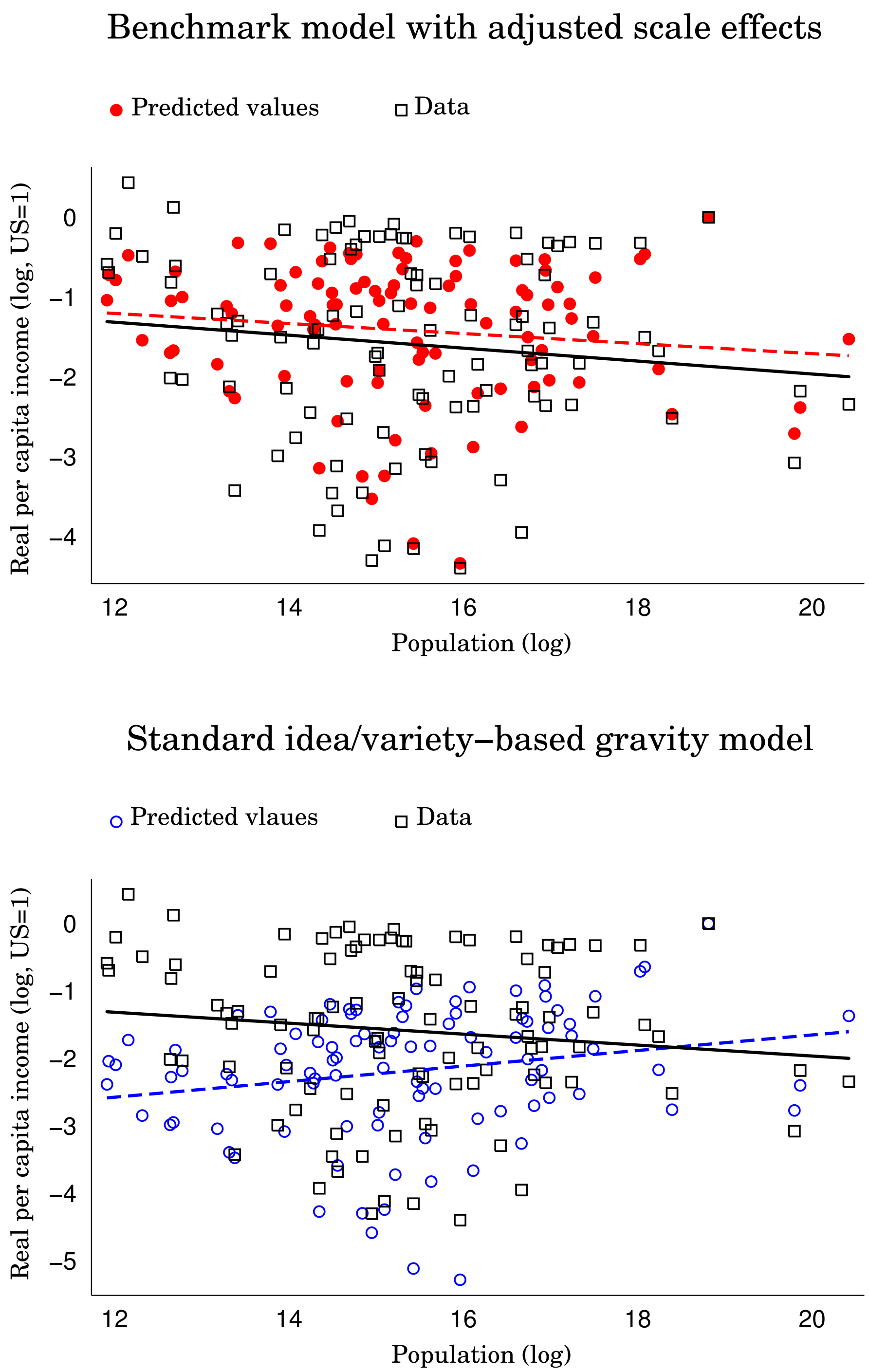
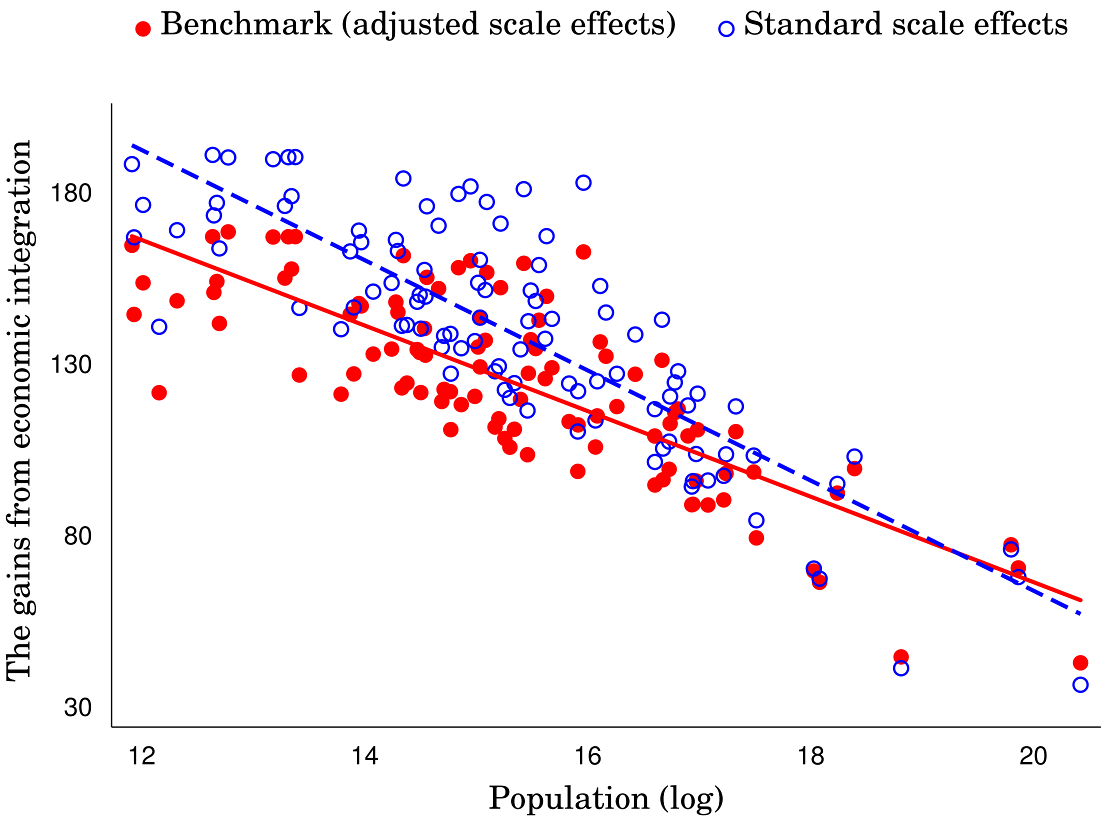


Figure 3: The gains from economic integration.



tions. It is well-known that the size of scale effects determine the optimal size of nations (see Alesina et al. (2000) and Alesina and Spolaore (2005)). In a standard trade model with unadjusted scale effects the returns to national size are non-diminishing. In such models political integration is isomorphic to economic integration. When ideas/products are nationally differentiated, the returns to national size are diminishing. Therefore, economic integration (through free trade) across politically independent nations is strictly superior to complete political integration. The intuition is that removing political borders (e.g. unifying the educational and political systems and the language) forces ideas and products to become more homogeneous. Economic integration, by contrast, permits the development and free exchange of highly differentiated products across political borders. To show this formally, consider a symmetric works with N countries, each with population L. Real per-capita income under economic integration will be

$$W_E = \gamma \left(N \times L^{\frac{\theta_2}{\theta_1}} \right)$$

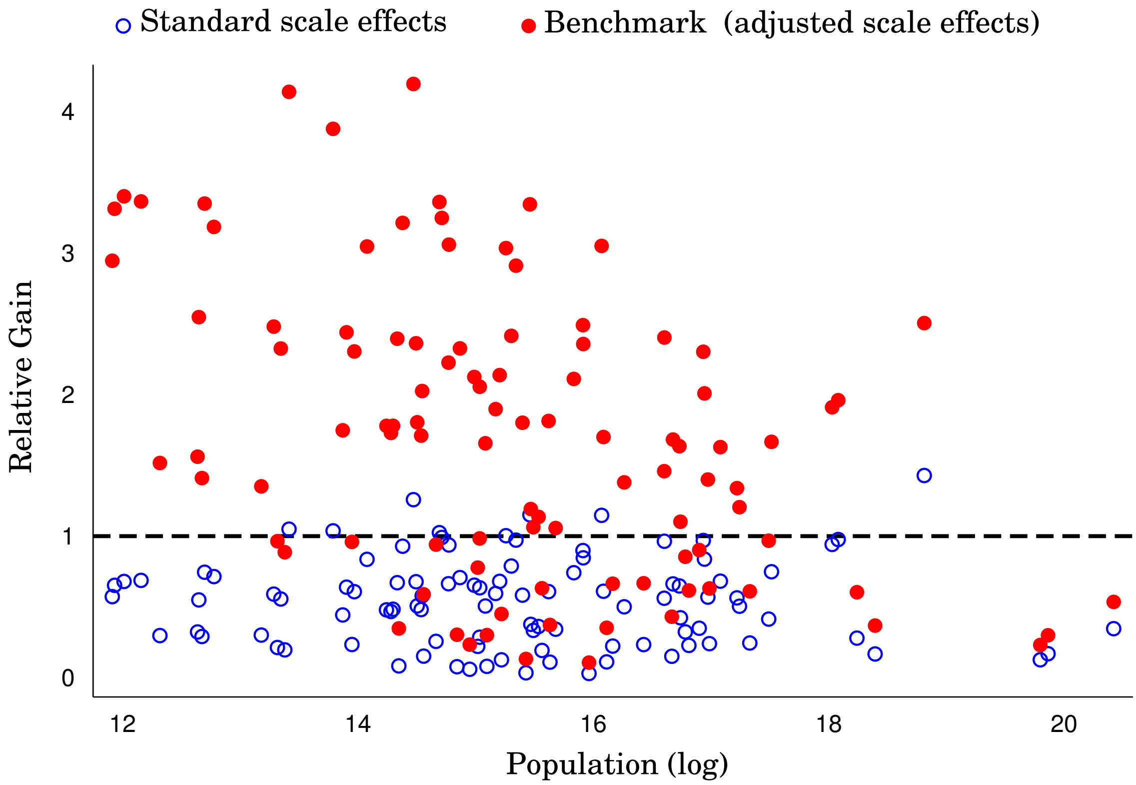
Under political integration we have on country with population $N \times L$ and a real per-capita income equal to

$$W_P = \gamma \left(N \times L \right)^{\frac{\theta_2}{\theta_1}}$$

Given that $N > 1$ and $\frac{\theta_2}{\theta_1} < 1$ it follows that $W_E > W_P$. We quantify we these effects by conducting two counterfactual analyses: first, we eliminate trade costs and then we eliminate political borders bundling all countries together.[10] Figure 4 plots the gains from economic integration relative to those from political integration, against population size. Unlike the standard trade model, our benchmark model implies that economic integration in a politically disintegrated world is 1.7 times superior to full political integration.

[10]The efficiency level in the politically integrated world is the population-weighted average of national efficiency levels.

Figure 4: *The gains from economic integration relative to the gains from political integration.*



6 Conclusion

We estimate that due to national product/idea differentiation, the degree of aggregate returns to scale are lower than those usually assumed in the trade literature. We also demonstrate that the degree of aggregate returns to scale vary tremendously across industries. These variations are a key determinant of the gains from trade. In theory, it is well understood that the gains from trade favor countries that enjoy comparative advantage in industries with stronger scale effects. Our estimates provide a foundation for quantifying these predictions, thus opening up an important avenue for future research. Furthermore, our estimates suggest that ideas and products are more homogenous within national borders. This finding has straightforward implications for a vast literature that studies the benefits of political integration and trading blocs.

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A Trade Shares in the Idea-Based Model

The discrete choice problem has the following formulation

$$\min_{\omega} \frac{p_\omega}{z_\omega(n)} \sim \max_{\omega} -\ln \frac{p_\omega}{z_\omega(n)} = -\ln p_\omega + \ln z_\omega(n)$$

Define $G(c)$ as follows

$$G(\mathbf{c}) = \sum_k \left(\sum_{\omega \in \Omega_k} \tilde{T}_k \exp(-\theta \ln p_\omega) \right)^{\frac{\theta_2}{\theta_1}} = \sum_k \left(\sum_{\omega \in \Omega_k} T_k^{\theta_1} p_\omega^{-\theta_1} \right)^{\frac{\theta_2}{\theta_1}} = \sum_k \left(\sum_{\omega \in \Omega_k} c_\omega^{\theta_1} \right)^{\frac{\theta_2}{\theta_1}}$$

where $\tilde{T}_k = T_k^{\theta_1}$ and $c_\omega \equiv T_k / p_\omega$. Note that $G(.)$ is a continuous and differentiable function of vector $\mathbf{c}$(where) and has the following properties:

i. $G(.) \geq 0$;

ii. $G(.)$ is a homogeneous function of rank θ_2: $G(\mathbf{c}) = \rho^{\theta_2} G(\mathbf{c})$;

iii. $\lim_{c_\omega \to \infty} G(\mathbf{c}) = \infty$, $\forall \omega$;

iv. the k'th partial derivative of $G(.)$ with respect to a generic combination of k variables c_ω, is non-negative if k is odd and non-positive if k is even.

Manski et al. (1981) show that if $G(.)$ satisfies the above conditions, and z_ω's are drawn from the following distribution:

$$F(z) = \exp\left(-G(e^{-\ln \mathbf{z}})\right) = \exp\left(-\sum_k \left(\sum_{\omega \in \Omega_k} \tilde{T}_k z_\omega^{-\theta_1} \right)^{\frac{\theta_2}{\theta_1}} \right)$$

the probability of choosing variety ω from country j is

$$\lambda_{\omega j} = \frac{\left(\frac{c_{\omega j}}{\theta_2}\right) \frac{\partial G}{\partial c_{\omega j}}}{G} = \frac{c_{\omega j} c_{\omega j}^{\theta - 1} \left(\sum_{\omega' \in \Omega_j} c_{\omega' j}^{\theta_1} \right)^{\frac{\theta_2}{\theta_1} - 1}}{\sum_k \left(\sum_{\omega' \in k} c_{\omega' k}^{\theta_1} \right)^{\frac{\theta_2}{\theta_1}}} = \frac{c_{\omega j}^{\theta}}{\sum_{\omega \in \Omega_j} c_{\omega' j}^{\theta_1}} \cdot \frac{\left(\sum_{\omega' \in \Omega_j} c_{\omega' j}^{\theta_1} \right)^{\frac{\theta_2}{\theta_1}}}{\sum_k \left(\sum_{\omega' \in k} c_{\omega' k}^{\theta_1} \right)^{\frac{\theta_2}{\theta_1}}}$$

Given that $c_{\omega j} = p_{\omega j}/T_j$ and defining $P_j \equiv \left[\sum_{\omega' \in \Omega_i} \left(p_{\omega' j}/T_j\right)^{-\theta_1}\right]^{-\frac{1}{\theta_1}}$ and $P \equiv \left[\sum P_k^{-\theta_2}\right]^{-\frac{1}{\theta_2}}$ we can write the above probability as:

$$\lambda_\omega = \left(\frac{p_{\omega j}/T_j}{P_j}\right)^{-\theta_1} \left(\frac{P_j}{P_i}\right)^{-\theta_2}$$