

# Computing Multilateral Resistances of Trading Countries

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## Abstract

Gravity Equations are broadly used to estimate the determinants of international trade flows. Empirical applications of the gravity equations are frequently criticized, because the estimated coefficients seem to be overestimated. This study builds on a theoretical derivation of the gravity equation provided by Anderson and van Wincoop (2003). They conclude that exports not only depend on bilateral trade costs, but also on bilateral trade costs relative to a measure of both countries' trade costs to all other countries, so called *multilateral resistances*. In this article I take a new theory-based index of comprehensive trade costs between two countries to compute these multilateral resistances. Then I use both the index for trade costs and the computed index for multilateral resistances to estimate the theoretically founded gravity equation and show how the values of the estimated coefficients melt.

*JEL classification:* F13, F17, C63, C33

*Keywords:* Gravity Equation, Multilateral Resistances, Endogenous Trade Costs

# 1 Background

In a general equilibrium framework with many countries trading composite goods that are differentiated by country of origin, Anderson and van Wincoop (2003) derive the following gravity equation:

$$X_{ij} = \frac{Y_i \cdot Y_j}{Y_w} \cdot \left( \frac{t_{ij}}{P_i \cdot P_j} \right)^{1-\sigma}. \quad (1)$$

Here,  $Y_i$  and  $Y_j$  are the exogenously given GDPs of the countries,  $Y_w$  the GDP of the whole world, and  $\sigma$  is the elasticity of substitution. It is assumed that  $\sigma > 1$ , which is supported by empirical evidence (see Anderson and van Wincoop, 2004). Trade costs in terms of iceberg trade costs are indicated by  $t_{ij} > 1$ . These iceberg cost can be interpreted as a tariff equivalent: selling a good from country  $i$  in country  $j$  raises the price on country  $j$ 's market by  $(t_{ij} - 1)\%$ . Note that this modeling of trade costs reflects per Dollar trade costs rather than total trade costs. This means that  $t_{ij}$  describes the average mark up of trade costs on each Dollar of transport value. Furthermore it is assumed that trade costs inside a country are benchmarked to one,  $t_{ii} = 1$ , and that transport costs between two countries are symmetric,  $t_{ij} = t_{ji}$ .<sup>1</sup>  $P_i$  and  $P_j$  denote the exogenously given multilateral resistances of the exporting or the importing country, respectively. They are derived from a Dixit-Stiglitz

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<sup>1</sup> This symmetric assumption could be relaxed, but because the empirical trade cost index introduced in section 2 is a geometrical mean of trade costs and thus a symmetric measure of trade costs, this assumption helps to simplify.

price index and can be written as:<sup>2</sup>

$$P_i = \left( \sum_j \left( \frac{t_{ij}}{P_j} \right)^{1-\sigma} \cdot s_j \right)^{1/(1-\sigma)}, \quad (2)$$

$$P_j = \left( \sum_i \left( \frac{t_{ij}}{P_i} \right)^{1-\sigma} \cdot s_i \right)^{1/(1-\sigma)}. \quad (3)$$

Multilateral resistances can be interpreted as an index for the overall accessibility to trade of a country. In the second multiplier of gravity equation (1), bilateral per Dollar trade costs  $t_{ij}$  appear in relation to the respective countries' multilateral resistances.<sup>3</sup> For illustration, imagine two countries lying isolated from the rest of the world on one island in the ocean, far away from the next continent. Bilateral average trade costs measured by iceberg-factor  $t_{ij}$  might be low and this should guarantee for a higher trade volume between both countries. But the relatively high trade costs to the complete rest of the world have an additional positive effect on the bilateral trade volume. If the same two island-countries were two small countries in the middle of a huge continent with many huge countries surrounding them, multilateral resistances were much lower and thus trade volume between the two countries

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<sup>2</sup> Note that Anderson and van Wincoop (2003) distinguish more precisely between multilateral resistances of exporting countries on the one hand and multilateral resistances importing countries on the other hand. If trade costs are assumed to be symmetric between all countries ( $t_{ij} = t_{ji}$ ), which is also a relevant assumption for this paper, it can be shown that the export multilateral resistance of a country equals the import multilateral resistance (Anderson and van Wincoop, 2003, p. 175).

<sup>3</sup> Notably, the effect of multilateral price indexes was already stated in the first theoretical derivations of the gravity equation (see Anderson, 1979; Bergstrand, 1985). But Anderson and van Wincoop (2003) concentrated this issue on the elegant formulation of equation (1) and were able to conclude that ignoring multilateral resistances leads to biased results.

were lower, even if for the GDPs and  $t_{ij}$  the same levels are chosen.

The aim of this paper is to find a computational solution for multilateral resistances. On its right hand side, the theory-based gravity equation (1) has a directly measurable part, containing the GDPs, and an indirectly measurable part, containing trade costs and multilateral resistances. The indirectly measurable part is usually complemented by replacing  $t_{ij}$  via proxy variables (like distance, exchange rate volatilities, membership in a certain country group and much more) and controlling multilateral resistances by fixed effect dummies (country or country-pair dummies). In this paper I replace the indirect method to consider bilateral trade costs by a novel index (Novy, 2007) that makes it possible to yield direct data for bilateral trade costs.

Recent work by Baier and Bergstrand (2009) pursues a similar aim. They use a Taylor-series expansion to solve for multilateral resistances. But this approach requires a normalization of the resistances to a reference country, so that each computed multilateral resistance must be interpreted relative to a certain country that has to be chosen in advance. In contrast, my approach is able to compute direct absolute values for the multilateral resistances. A normalization to a certain country is not necessary.

The paper is structured as follows. The calculation of the index for bilateral trade cost is briefly described in section 2. The presence of direct data for bilateral trade costs makes it possible to solve the multilateral resistances. A procedure to do so is presented in section 3. With measures for trade costs and multilateral resistances it becomes possible to estimate the gravity equation (1) directly. The econometric model, the used data and the results are described in section 4. Section 5 concludes.

## 2 Computing average trade costs

Building on the theoretical framework of the gravity equation introduced by Anderson and van Wincoop (2003), Novy (2007) derives an index for the geometric mean of the bilateral trade costs between two countries:

$$t_{ij} = \left( \frac{X_{ii}X_{jj}}{X_{ij}X_{ji}} \right)^{\frac{1}{2(\sigma-1)}}. \quad (4)$$

In this index trade barriers between two countries are a function of the ratio between intra-national trade ( $X_{ii}, X_{jj}$ ) and international trade ( $X_{ij}, X_{ji}$ ). The higher the trade inside a country relative to the exports to the other country, the higher are the bilateral trade costs, since  $\sigma$  is assumed to be larger than 1. Note that this index is a comprehensive measure of trade costs. These comprehensive trade costs can be decomposed into measurable components and not measurable components.<sup>4</sup>

Because there are many sources to access the necessary data, equation (4) makes it feasible to compute a theory-based index for the overall trade costs between two countries. In this paper, I use a set of 23 OECD countries for the years 1995 to 2005.<sup>5</sup> The data source for bilateral exports is the bilateral trade statistics of OECD's Structural Analysis (OECD STAN). Following

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<sup>4</sup> See Anderson and van Wincoop (2004) for a comprehensive discussion of trade costs. They decompose overall trade costs into three classes: transport costs, border related costs and retail/wholesale costs.

<sup>5</sup> The countries are selected so that the full data for the considered period is available. This is necessary to make multilateral resistances comparable over time. The countries covered by the data set are Australia, Austria, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom and the United States.

Novy (2007), intra-national trade flows are computed as a country's total production minus total exports. If it is available, the production data is taken from the OECD STAN data (converted into US Dollars using the OECD Financial Indicators annual exchange rates). Since there are many missing observations (e.g. Turkey is completely not reported in this data set), I compensate for the missing data by using data from the World Development Indicators 2008.<sup>6</sup>

### 3 Computing the Multilateral Resistances

Multilateral resistances, as they are given by equations (2) and (3), become computable if there is data for the world GDP shares of the countries  $i$  and  $j$ ,  $s_i$  and  $s_j$ , as well as for bilateral trade costs  $t_{ij}$ . While GDP share data is directly available from several data sources (like OECD STAN), trade costs can be measured by the index presented in the previous section. Therefore it is possible to compute multilateral resistances by solving the equation system given by equations (2) or (3), respectively.

#### 3.1 The equation system and its solution

To understand the algebraic structure of the multilateral resistance index, it is useful to bring (2) in a form that shows the equation for each single

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<sup>6</sup> In appendix A this procedure is described in detail.



study includes 23 countries, which makes it useful to construct an alternative approach that finds only the relevant solution of the equation system.

The idea behind this algorithm is simple. An equation system is solved if the left hand side equals the right hand side for each equation after inserting numerical values for the unknowns. If we choose certain values for each  $\Pi$  on the right hand side of equation system (5) which yield the same values for each corresponding  $\Pi$  on the left hand side, these chosen values must be a solution of the equation system. The pursued method to find these values works as follows. Using equation system (5), we choose one singular value for all the  $\Pi_j$  on the right hand side, call it  $\Pi_{(0)}$ , and compute  $\Pi_{i(1)} = \left(\frac{1}{\Pi_{(0)}}\right) \cdot \left(\sum_j \theta_{ij} \cdot s_j\right)$  in a first round. Note that the value of  $\Pi_{(0)}$  is the same for each country  $j$ , meaning that the start value is independent the country. Then we use the resulting  $\Pi_{i(1)}$ -vector from this calculation to compute  $\Pi_{j(2)} = \left(\sum_i \left(\frac{1}{\Pi_{i(1)}}\right) \cdot \theta_{ij} s_i\right)$  in a second round. This procedure is repeated until each  $\Pi_i$  converges, meaning that there are no (or negligibly little) changes after several repeated recalculation rounds. In this case the value of each  $\Pi$  on the right hand side equals the value of the corresponding  $\Pi$  on the left hand side: we yield a certain value for each  $\Pi_i$  on the left hand side that is equal to all  $\Pi_i$ s plugged in on the right hand side equations. This must be one solution of the equation system.

How do we choose the right value,  $\Pi_{(0)}^*$ ? Running the recalculation of the data sample with too small values of  $\Pi_{(0)}$  leads to an alternating sequence: the results of the odd rounds of recalculation are too low, the results of the even rounds are too high and so on. Running the recalculation of the data sample with too high values for  $\Pi_{(0)}$  leads to an adverse alternating sequence, where the odd recalculation rounds yield too high and the even recalculations too



low results. The closer  $\Pi_{(0)}$  is to the optimal starting value  $\Pi_{(0)}^*$ , the smaller is the amplitude of the recalculated values.

### 3.2 An illustrative example

An example shall help to understand this procedure. With some tricks, a set of three polynomial equations can easily be transformed into a square linear equation system which can be solved with Cramer's Rule. This direct solution is a reliable benchmark for the results from the numerical procedure. Although the assumptions made to get the square linear system are not in line with the definition of multilateral resistances, this example might help to understand the mechanics of solving the equations.

Starting from equation system (6), define the unknown  $z_{ij} = z_i \cdot z_j$  and multiply each summand in the parentheses on the right hand side of (6) with  $z_i$  for the case  $C = 3$ , to get:

$$\begin{aligned} 1 &= z_{11}b_{11} + z_{12}b_{12} + z_{13}b_{13}, \\ 1 &= z_{21}b_{21} + z_{22}b_{22} + z_{23}b_{23}, \\ 1 &= z_{31}b_{31} + z_{32}b_{32} + z_{33}b_{33}. \end{aligned} \tag{7}$$

Assume that  $b_{ii} = 0$ . Note that this assumption does not adequately reflect the definition of multilateral resistances, but is necessary to get a symmetric linear equation system. Following the assumptions of the economic model, each  $b_{ii}$  is strict greater than 0 because  $t_{ii} = 1$ ,  $\sigma > 1$  and  $0 < s_i < 1$ . More precisely, this assumption ignores country  $i$  itself in the summation of all countries to compute multilateral resistances as given by equation (2).<sup>7</sup>

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<sup>7</sup> Equation (2) becomes:

Since  $z_{ij} = z_{ji}$ , because  $1/(\Pi_i\Pi_j) = 1/(\Pi_j\Pi_i)$ , it becomes possible to rewrite equation system (7) into a square linear equation system:

$$\begin{aligned} 1 &= z_{12}b_{12} + z_{13}b_{13} + 0, \\ 1 &= z_{12}b_{21} + 0 + z_{23}b_{23}, \\ 1 &= 0 + z_{13}b_{31} + z_{23}b_{32}, \end{aligned} \tag{8}$$

or in matrix form:

$$\underbrace{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}}_{\mathbf{1}} = \underbrace{\begin{pmatrix} b_{12} & b_{13} & 0 \\ b_{21} & 0 & b_{23} \\ 0 & b_{31} & b_{32} \end{pmatrix}}_{\mathbf{B}} \cdot \underbrace{\begin{pmatrix} z_{12} \\ z_{13} \\ z_{23} \end{pmatrix}}_{\mathbf{z}}. \tag{9}$$

Using Cramer's Rule, this square linear equation system can easily be solved for the three unknowns ( $z_{12}^*$ ,  $z_{13}^*$ ,  $z_{23}^*$ ):

$$z_{12}^* = \frac{1}{\Pi_1\Pi_2} = \frac{-b_{13}b_{23} + b_{23}b_{31} + b_{13}b_{32}}{b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32}}, \tag{10}$$

$$z_{13}^* = \frac{1}{\Pi_1\Pi_3} = \frac{b_{12}b_{23} - b_{12}b_{32} + b_{21}b_{32}}{b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32}}, \tag{11}$$

$$z_{23}^* = \frac{1}{\Pi_2\Pi_3} = \frac{b_{13}b_{21} + b_{12}b_{31} - b_{21}b_{31}}{b_{12}b_{23}b_{31} + b_{13}b_{21}b_{32}}. \tag{12}$$

From these solutions it is possible to compute the values for the wanted multilateral resistances. Solving the system  $z_{ij} = 1/(\Pi_i\Pi_j)$  for each  $\Pi_i$  with

$$P_i = \left( \sum_{j \neq i} \left( \frac{t_{ij}}{P_j} \right)^{1-\sigma} \cdot s_j \right)^{1/(1-\sigma)} \text{ instead of } \left( \sum_j \left( \frac{t_{ij}}{P_j} \right)^{1-\sigma} \cdot s_j \right)^{1/(1-\sigma)}.$$

The general theory-based case of transforming the polynomial equation system into a linear equation system is presented in appendix B.

$i, j \in \{1, 2, 3\}$  yields

$$P_1 = \Pi_1^{1/(1-\sigma)} = \left( \sqrt{\frac{z_{23}^*}{z_{12}^* z_{13}^*}} \right)^{1/(1-\sigma)}, \quad (13)$$

$$P_2 = \Pi_2^{1/(1-\sigma)} = \left( \sqrt{\frac{z_{13}^*}{z_{12}^* z_{23}^*}} \right)^{1/(1-\sigma)}, \quad (14)$$

$$P_3 = \Pi_3^{1/(1-\sigma)} = \left( \sqrt{\frac{z_{12}^*}{z_{13}^* z_{23}^*}} \right)^{1/(1-\sigma)}. \quad (15)$$

Note that a real solution for the multilateral resistances can only be obtained, if there is no negative  $z_{ij}^*$ . This condition depends on the values of the  $b_{ij}$ -coefficients:  $t_{ij}$ ,  $s_j$  and  $\sigma$ .<sup>8</sup>

Now, we consider a numerical example for the three country model. First we solve the equations using Cramer's Rule to get a benchmark and then we apply the numerical algorithm. We use the data for countries 1, 2 and 3 of table 1 as given with an elasticity of substitution  $\sigma = 8$  and remember that the impact of a country on its own multilateral resistances is ignored by assumption ( $t_{ii} = 0$ ) to provide a special case, where a simple solution of the equation system is possible. Country 1 is the biggest country and has the lowest trade costs. So we expect a low multilateral resistance. For country 3, the opposite should be the case.

We directly solve the multilateral resistances to obtain a reliable reference. The values for the coefficients  $b_{ij} = t_{ij}^{-7} s_j$  can be directly computed from table 1. Applying equations (10) to (12) yields the solution of the equation

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<sup>8</sup> All four determinants used for Cramer's Rule, the main determinant and the three column-replaced determinants, must be greater than 0, or all of them must be smaller than 0.

Country	1	2	3
GDP-share $s_i$	0.4%	0.35%	0.25%
	Trade Costs $t_{ij}$		
1	0	1.2	1.3
2	1.2	0	1.4
3	1.3	1.4	0

Table 1: Assumed data for the numerical example

system (9):

$$z_{12}^* = 6.399, \quad (16)$$

$$z_{13}^* = 9.412, \quad (17)$$

$$z_{23}^* = 12.047. \quad (18)$$

Because no  $z_{ij}^*$  is negative, it is possible to find real positive solutions for the multilateral resistances by applying equations (13) to (15):

$$P_1 = 1.122 \quad (19)$$

$$P_2 = 1.162 \quad (20)$$

$$P_3 = 1.228 \quad (21)$$

Does the algorithm yield the same numbers for multilateral resistances? Under the assumptions and with the numbers of table 1, we can write equation system (5):

$$\begin{aligned} \Pi_A &= 0 + \frac{1}{\Pi_B} \cdot 0.098 + \frac{1}{\Pi_C} \cdot 0.040, \\ \Pi_B &= \frac{1}{\Pi_A} \cdot 0.112 + 0 + \frac{1}{\Pi_C} \cdot 0.024, \\ \Pi_C &= \frac{1}{\Pi_A} \cdot 0.064 + \frac{1}{\Pi_B} \cdot 0.033 + 0, \end{aligned} \quad (22)$$

Now we choose a common value for all multilateral resistances,  $P_{(0)} = \Pi_{1(0)}^{-7} = \Pi_{2(0)}^{-7} = \Pi_{3(0)}^{-7}$ , insert this value for all the  $\Pi$ 's on the right hand side of equation system (22) to obtain the values for the  $\Pi$ 's on the left hand side,  $\Pi_{1(1)}, \Pi_{2(1)}$  and  $\Pi_{3(1)}$ . In a further round, these values are used on the right hand side to compute  $\Pi_{1(2)}, \Pi_{2(2)}, \Pi_{3(2)}$  and so on. The results for different start values  $P_{(0)}$  are reported in table 2.

Four important outcomes can immediately be seen from the numbers of this table. First, where the start value  $P_{(0)}$  is 1.15 (third line), differences between the single rounds of recalculation are relatively small, compared to the other results. These differences grow both if the value for  $P_{(0)}$  shrinks to 1.00 or raises to 1.50. Second, if starting values are lower than 1.15 (line 1 and 2), results of an odd recalculation round are lower than those of an even one. If a starting value larger than 1.15 is used (line 4 and 5), the opposite is the case: the values after e.g. the 99th recalculation are always higher than the values after 100 recalculations. Third, after many recalculations the results alternate around singular values of  $P_1, P_2$  and  $P_3$ . The value after 97 recalculations is the same as after 99 recalculations, the value after 98 is the same as after 100 recalculations. And fourth, after a sufficiently high number of recalculations, the wanted values given by the direct solution of the equation (19) to (21) lie always between the maximum and minimum values of the alternating sequences.

Is it possible to find a start value  $P_{(0)}^*$ , where no differences between the single recalculation rounds appear anymore? To face this problem, a kind of search algorithm is used. Start for example with  $P_{(0)} = 1.00$ . The values after 100 recalculations are larger than those after 99 recalculations. Repeat the procedure with  $P_{(0)} = 1.10$  and then with  $P_{(0)} = 1.20$ . In the latter case,

the altering sequence changes: values after 100 recalculations are lower than after 99. Now go down in 0.01 steps until the structure changes at 1.15, go up in 0.001 steps and stop when the changes between recalculation 99 and 100 are small enough, measured e.g. by the sum of differences between the values in the two last undertaken recalculation rounds. This procedure leads to an optimal start value  $P_{(0)}^* \approx 1.158672$ . The more exact this start value is chosen, the closer the results of the numerical simulation fit the results of equations (19) to (21).

Figures 1 and 2 finally illustrate the procedure graphically. Figure 1 starts with values  $P_{(0)} < P_{(0)}^*$ . The amplitude first goes down, then up. Figure 2 with values  $P_{(0)} > P_{(0)}^*$ . The amplitude first goes up, then down. In both cases, more distance to  $P_{(0)}^*$  increases the amplitude between the recalculation results (light gray and gray alternating sequences). Choosing the optimal value  $P_{(0)}^*$  leads to a steady course of the three sequences (black bold sequence). These sequences with the optimal values converge to the solutions of the equation system.

### 3.3 Multilateral Resistances of OECD-countries

The numerical procedure is applied on the data set with 23 OECD Countries. Because this panel data set comprises data of 11 years, it is necessary to compute the multilateral resistances separately for each year. That means, we must separate the data by year and find 11 different start values. The algorithm to find these start values follows the same idea as described in the example: start with 1, go up in 1 steps until the amplitude changes, then go

down in 0.1 steps until the amplitude changes, then go up in 0.01 steps and so on.<sup>9</sup> The *start value* of the algorithm is 1. The first *step size* is 1. After each change of the amplitude, the step size is set on one tenth of the step size before. The *number of recalculations* with a certain start value is 100.<sup>10</sup>

It is necessary to choose a condition when the convergence is sufficient and the program stops searching the optimal start value. As a measure of sufficiency I choose the sum of all differences between the last and the penultimate recalculation round. So we take the differences of all observations between the 100st and 99th recalculation round and sum it up. If this sum is lower than  $\pm 10^{-6}$  the accuracy of the simulated values is considered to be sufficiently high. For some years a marginal amplitude remains. In these cases, the algorithm is stopped when the step size arrives a value of  $10^{-9}$ .

Are these conditions for breaking the algorithm adequate? There are two important requests on the simulation: (i) the amplitude, meaning the difference between odd and even recalculation rounds, should be zero and (ii) the simulation must converge after less than 100 recalculations. Table 3 reports some descriptive statistics over all 5,566 observations<sup>11</sup> of the differences between the last (100st) and certain recalculation rounds before with the approximately optimal start value  $P_{(0)}^*$  resulting from the search algorithm

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<sup>9</sup> This algorithm is surely not the most efficient one. For larger data sets more advanced programming efforts should be applied to minimize the runtime of the program.

<sup>10</sup> To check the robustness of the simulation, the number of recalculations was extended up to 150. The results remain exactly the same. The results also remain robust if other start values or step sizes are used.

<sup>11</sup> 23 countries  $\times$  22 trade partners  $\times$  11 years = 5,566 observations.

described above. The summary statistics of these differences between each single realization of the 100st the 99th recalculation round ( $\Delta_1 \equiv P_{i100} - P_{i99}$ ) are reported in the first line. If these differences differ from zero or a sufficiently low number, there is still an amplitude and the start value found by the algorithm is not yet adequate. As can be seen, mean and standard deviation are not zero. But the values are very low: they become relevant behind the sixth decimal place. So if these deviations from zero are due to a remaining amplitude, they are so small that they can be neglected. It is worthwhile to take a closer look at this amplitude. Table 4 shows that there are only four values taken by  $\Delta_1$  over the whole 5,566 observations:  $-2.38 \cdot 10^{-7}$ ,  $-1.19 \cdot 10^{-7}$ , 0 and  $+1.19 \cdot 10^{-7}$ . 63% of the observations take exactly the value 0, the relatively “large” amplitude of  $-2.38 \cdot 10^{-7}$  affects only 4% of the observations. Because  $-2.38 \cdot 10^{-7}$  is two times  $-1.19 \cdot 10^{-7}$  which is the negative value of  $+1.19 \cdot 10^{-7}$ , it is not unlikely that this bias results from a computational problem like a systematic rounding error caused by the used software. The second line of table 3 shows that over the last 9 recalculation rounds these differences are the same. This means that the amplitude is constant at least over the last 9 recalculation rounds. Analyzing the amplitude yields the same results as described in table 4. The two last lines of table 3 show that there is no change reported for the 5,566 realizations of the multilateral resistances over the last 2 and the last 10 recalculation rounds:  $\Delta_2$  and  $\Delta_{10}$  are exactly reported as zero. This implies that the values have (i) a sufficiently low amplitude and that (ii) the values have attained their full convergence after less than 90 rounds of recalculation.

Table 5 describes the derived multilateral resistance data for the 23 countries of the data set, comprised over the eleven years of observation. The country



with the lowest multilateral resistances is Canada. This result does not surprise, because trade costs between Canada and the United States are very low. The United States is the biggest economy in the set which guarantees for a high weight ( $s_j$  around 40%) in the summation. They have relatively high multilateral resistances. From the definition of multilateral resistances given by equation (2) or (3) it becomes obvious that multilateral resistances of a country must be low, if this country has extremely low trade costs with an extremely large country that has high multilateral resistances. Further countries with low multilateral resistances are the Netherlands, Germany, Ireland and the United Kingdom. Countries with high multilateral resistances are the mediterrian countries Greece, Portugal and Turkey, as well as the former Eastern Block countries Poland and Hungary. Note that Poland, Turkey and especially Hungary were able to decrease their multilateral resistances over the period from 1995 to 2005, while the level of Greece did not change over this time and Portugal even raised its multilateral resistances. Another country with increasing multilateral resistances is the United States which might be caused by the terrorist attacks of September 11th 2001. Figures 3 to 5 show the development of the multilateral resistances of some OECD countries over the time period from 1995 to 2005.

## 4 Estimation of the Theory Based Gravity Equation

The availability of data for average trade costs ( $t_{ij}$ ) and multilateral resistances ( $P_i$  and  $P_j$ ) makes it possible to estimate a log-linear form of the

theory-based gravity equation (1). Since there is evidence that per Dollar trade costs are endogenously affected by policy variables as well as natural trade cost barriers, and that they are reversely connected to bilateral export volumes due to economies of scale in the trade sector, direct estimates of equation (1) would be biased (see Rudolph, 2009a,b).

## 4.1 Econometric Model and Data

The standard approach of estimating the gravity equation is:

$$\text{exp}_{ij} = \pi_0 + \pi_1 \text{gdp}_i + \pi_2 \text{gdp}_j + \pi_3 \text{pi} + \pi_4 \text{pj} + \sum_{k=5}^{20} \pi_k w_{ij}^k + \epsilon_{ijt}, \quad (23)$$

where  $\text{exp}_{ij}$  is the log of bilateral exports,  $\text{gdp}_i$  and  $\text{gdp}_j$  are the logs of the exporting and importing country's GDP. The exporting country is always denoted by  $i$ , the importing country by  $j$ . Data source for exports and the GDPs is the OECD Structural Analysis Data Base (OECD STAN). The logs of the exporting or importing country's multilateral resistances, as they are computed in section 3, are denoted by  $\text{pi}$  and  $\text{pj}$ . These values result from the calculations presented in the previous sections. The vector  $w_{ij}^k$  concludes the following trade cost proxies: freedom of trade index by the Bell Foundation ( $\text{trfi}$  and  $\text{trfj}$ ), geographic distance between the trading countries in logs ( $\text{dist}$ ), exchange rate volatility in logs ( $\text{exvol}$ ), dummies for common language ( $\text{lang}$ ), common border ( $\text{bor}$ ), EU membership of the exporting or importing country ( $\text{eui}$  and  $\text{euj}$ ), continental location without access to the sea ( $\text{landli}$  and  $\text{landlj}$ ), location on an island ( $\text{isli}$  and  $\text{islj}$ ), membership in the commonwealth of nations ( $\text{cwni}$  and  $\text{cwnj}$ ) and former eastern block ( $\text{ebli}$  and  $\text{eblj}$ ).

The data set includes 23 OECD countries over the period from 1995 to 2005. Estimating this panel data set requires certain techniques to control for the effects of the countries and the years. Therefore, three specifications will be reported: (i) a pooled regression, where the panel data properties are ignored, (ii) a least square dummy variable (LSDV) model with 23 dummies for the exporting countries plus 23 dummies for the importing countries plus 11 dummies for the years (following e.g. Mátyás, 1997; Anderson and van Wincoop, 2003), and (iii) a LSDV model with  $23 \times 22 = 506$  country pair dummies plus 11 year dummies (see Cheng and Wall, 1999; Baltagi, Egger, and Pfaffermayr, 2003, for a discussion of the adequate panel specification of gravity equations).

Since we have constructed data for bilateral trade costs and multilateral resistances, the estimation of the standard gravity specification is not adequately based on the theory of Anderson and van Wincoop (2003). One problem of estimating the theory-based gravity equation (1) directly is that trade costs are exogenous. Changes in policy variables like freedom of trade or membership in a group of countries like the EU do not affect export levels between two countries directly. But they affect trade costs between the two countries directly and changes of those bilateral trade costs affect bilateral trade volumes. Ignoring this endogeneity of trade costs may lead to biased estimates. Following Rudolph (2009b) I also estimate the simultaneous equation model:

$$\text{exp}_{ij} = \alpha_0 + \alpha_1 \text{gdpi} + \alpha_2 \text{gdpj} + \alpha_3 \text{tij} + \alpha_4 \text{pi} + \alpha_5 \text{pj} + u_{ijt}, \quad (24)$$

$$\text{tij} = b_0 + b_1 \text{exp}_{ij} + b_2 \text{pi} + b_3 \text{pj} + \sum_{k=4}^{19} b_k w_{ij}^k + v_{ijt}, \quad (25)$$

with the trade cost index  $\text{tij}$  introduced in 2. To study the impact of introducing the multilateral resistances into the equation system, I first estimate

both equations (24) and (25) without multilateral resistances, second only equation (24) with multilateral resistances<sup>12</sup> and third both equations with multilateral resistances.

## 4.2 Empirical Results

Table 6 presents the results of estimating the standard gravity equation (23). The first two columns show the pooled regression that ignores the presence of panel data, columns (3) and (4) the country-year fixed effects model and columns (5) and (6) the country-pair-year fixed effects model. Columns (1), (3) and (5) display the reference case, where the effect of the multilateral resistances is assumed to be zero (or assumed to be completely captured by the fixed effects, respectively). In the results of columns (2), (4) and (6) the effects of the computed multilateral resistances are contained. An analysis of the residuals shows that in the case of the country-pair-year fixed effects model the residuals are closer to zero and that they are distributed independently from the endogenous variable  $\text{exp}_{ij}$  in comparison to the other specifications (figures 6 and 7). This indicates that the model with the country-pair-year fixed effects has the best properties to fit the model. The estimated coefficients of the multilateral resistances affect the exports negatively. Especially the coefficient of the exporting country's multilateral resistances has a high effect on trade flows. Note that the negative sign of these effects is not in line with the theory of Anderson and van Wincoop (2003), described in section 1. Controlling for the multilateral resistance index lowers the es-

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<sup>12</sup>In the theory proposed by Anderson and van Wincoop (2003), multilateral resistances only enter the gravity equation but not the trade cost equation.

estimated effects of the other exogenous variables, as can be seen immediately from the comparison of the results in columns (5) and (6).

Again, note that the standard gravity model does not exactly reflect the theory presented above, since constructed data for bilateral trade costs and multilateral resistances is available. Table 7 shows the results of the theory-based simultaneous equation model with country-pair-year fixed effects using a 2SLS estimator.<sup>13</sup> As a reference case, column (1) displays the results without multilateral resistances. In column (2) it is assumed that multilateral resistances affect the gravity equation (24), but not the trade cost equation (25). In column (3), it is additionally assumed that bilateral trade costs depend on multilateral resistances. Multilateral resistances have a highly significant effect on both exports and trade costs. Comparing columns (1), (2) and (3) shows that introducing multilateral resistances lowers the estimated coefficients in both simultaneous equations.

Note that the multilateral resistances of the importing country ( $p_j$ ) foster trade while the multilateral resistances of the exporting country ( $p_i$ ) enhance trade in this specification (upper part of table 7). Because the theory of Anderson and van Wincoop (2003) postulates that the effect of trade costs  $t_{ij}$  on exports must be seen in relation to multilateral resistances  $p_i$  and  $p_j$ , we would expect a positive sign for both coefficients and not only for the import country coefficient. To adjust the empirical model to the theory of equation

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<sup>13</sup>Using a 3SLS estimator for the country-pair-year fixed effects specification with its 517 dummy variables was not feasible. Table 7 only presents the results of a country-pair-year fixed effects estimation because this specification has the best fit of the model compared to pooled regression and country-year fixed effects. The results of the other specifications are available on request.

(1), I comprise  $p_i$  and  $p_j$  to  $p_{ij} = p_i \cdot p_j$ . The results of this restricted model are shown in columns (4) and (5). Here, the coefficient of the multilateral resistances' product has a highly significant impact on both, the exports via the gravity equation and the trade costs via the trade cost function. If the product of multilateral resistances (the trade barriers of two certain countries to all countries) increases and everything else (explicitly the trade costs between the two countries) is kept constant, exports between these two certain countries increase, because it becomes relatively more expensive for both countries to trade with the rest of the world than with each other. This is exactly the logic of the multilateral resistances introduced by Anderson and van Wincoop (2003). On the side of the trade costs, we find that there is a positive relationship between trade costs to all countries and trade costs to another country. If the product of two countries' multilateral resistances is high, this is positively correlated with the "bilateral resistances" meaning the bilateral trade costs of these two countries. The coefficients of the remaining exogenous variables do hardly differ from the model with unrestricted multilateral resistances. Controlling directly for comprehensive trade costs and multilateral resistances clearly reduces the estimated effects of the other exogenous variables in the model.

## 5 Conclusion

In their theoretical foundation of the gravity equation, Anderson and van Wincoop (2003) found that trade costs in gravity equations must be seen relatively to the trading countries multilateral resistances that reflect the countries' trade barriers to all other countries in the model. Neglecting this

issue leads normally to (normally upward) biased estimates. It became commonplace in the empirical literature applying the gravity equation to control for multilateral resistances by country or country-pair dummies.

This paper shows a way to quantify multilateral resistances. With an index for comprehensive bilateral trade costs, proposed by Novy (2007), it becomes possible to solve the equation system that defines multilateral resistances. Since a direct solution is not possible or feasible, respectively, a numerical procedure has been developed to compute multilateral resistances. The idea of this procedure is to find an optimal common start value for all countries' multilateral resistances, so that the equation system converges after repeated recalculations. The procedure works with OECD data. For all 11 calculations (for 11 years) the equation systems converge. The calculated values of the multilateral resistances are plausible.

Values for trade costs and multilateral resistances make it possible to estimate the theory-based gravity equation by Anderson and van Wincoop (2003) directly. Since trade costs should be endogenous and also depend on the bilateral exports they explain, the estimation should be done with a simultaneous equation model. The results of this estimation show that the computed multilateral resistances have a significant influence on both, bilateral exports and bilateral trade costs. It also appears that multilateral resistances clearly reduce the estimated effects of the other exogenous variables in the model.

## A Construction of the Trade Cost Index – Details

The trade volume inside a country ( $X_{ii}$  and  $X_{jj}$ ), or the domestic trade, can be interpreted as the country's production minus the sum of the exports into all countries. Since export data is measured in gross shipments while GDP data is based on value added and, additionally, contains services that are not considered in the export data, GDP is not suitable to calculate this index. Instead, following Wei (1996) and Novy (2007), production data for goods extracted from the OECD STAN Database is used and converted in US Dollars using the OECD Financial Indicators annual exchange rates. Unfortunately, production data is not available for some countries. Furthermore, Turkey is not considered in this data set at all. Therefore, missing values of production were constructed over the following three steps.

In a first step, I assume that in countries with higher productivity (measured by per-capita-income, source: World Development Indicators, WDI, 2008) the relation value added to production is higher. So, I calculate the elasticity of the value added/production-ratio with respect to per-capita-income using ordinary least squares. In a second step, I compute the missing data points from the estimated values of this regression if there is no data for production, but data for value added in the OECD data.

There are still many missing data points and Turkey is still omitted from the data. So, in a third step, I take the value added data from the World Development Indicators 2008 and, using an adjusted regression (intercept = 0) between OECD and WDI data, I find that OECD data systematically is



95% of the WDI values. Consequently, I multiply WDI data for value added by factor 0.95 and pursue the same procedure as in the first and the second step to compute missing production estimates for the case that there is no value added data available in the OECD STAN database, but in the WDI database.

Another crucial issue is the elasticity of substitution between the countries' composite goods,  $\sigma$ . In a survey of the empirical literature, Anderson and van Wincoop (2004) find that this elasticity takes values between 5 and 10. Thus, following Novy (2007), the elasticity of substitution is set  $\sigma = 8$ .<sup>14</sup> With the data and the assumption about  $\sigma$ , the logs of trade cost index,  $t_{ij}$ , can be computed.

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<sup>14</sup>A sensitivity analysis with  $\sigma = 5$  and  $\sigma = 10$  leads to exactly the same results.



## C Numerical Results of the Three-Country-Example

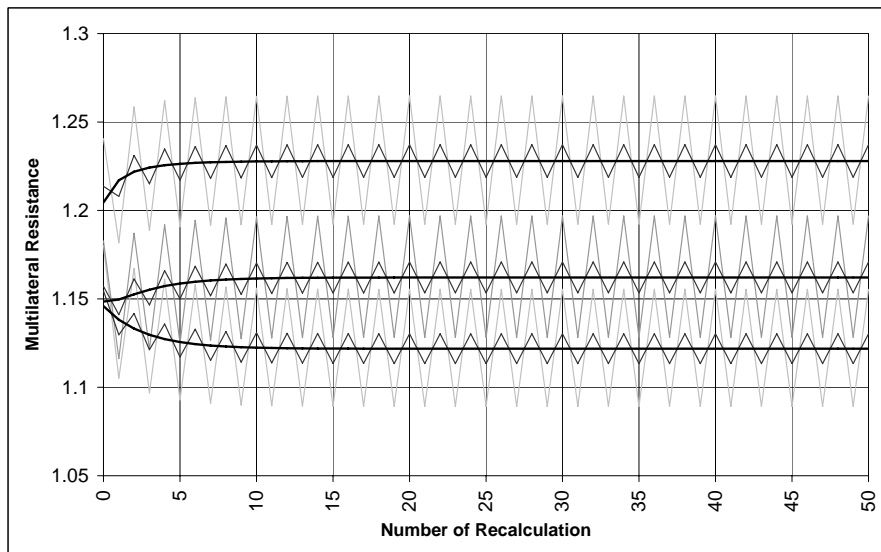


Figure 1: Simulation Results for the three countries' multilateral resistances, upward approximation:  $P_{(0)} = 1.125$  (light gray, large amplitude),  $P_{(0)} = 1.15$  (gray, small amplitude),  $P_{(0)} = P_{(0)}^* = 1.158672$  (black, steady course).

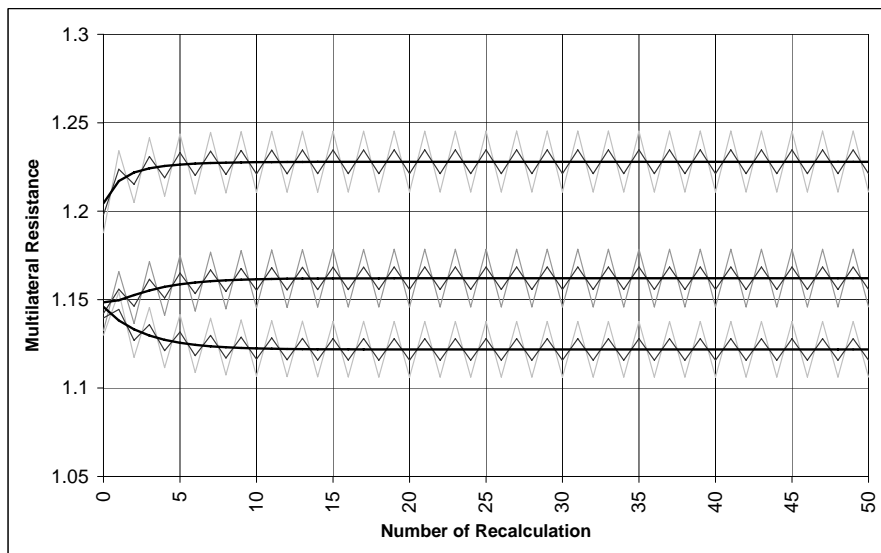


Figure 2: Simulation Results for the three countries' multilateral resistances, downward approximation:  $P_{(0)} = 1.175$  (light gray, large amplitude),  $P_{(0)} = 1.165$  (gray, small amplitude),  $P_{(0)} = P_{(0)}^* = 1.158672$  (black, steady course).

$P_{(0)}$	0	1	2	3	4	5	6	...	97	98	99	100
$P_A$	1.328	0.982	1.313	0.975	1.306	0.971	1.302		0.968	1.300	0.968	1.300
$P_B$	1.331	0.992	1.335	0.997	1.341	1.000	1.344	...	1.003	1.346	1.003	1.346
$P_C$	1.396	1.050	1.415	1.057	1.420	1.058	1.422		1.060	1.423	1.060	1.423
$P_A$	1.207	1.081	1.194	1.072	1.187	1.069	1.184		1.065	1.182	1.065	1.182
$P_B$	1.210	1.091	1.214	1.097	1.219	1.100	1.222	...	1.103	1.224	1.103	1.224
$P_C$	1.269	1.155	1.287	1.162	1.291	1.164	1.292		1.166	1.293	1.166	1.293
$P_A$	1.154	1.130	1.142	1.121	1.136	1.117	1.133		1.113	1.130	1.113	1.130
$P_B$	1.157	1.141	1.161	1.147	1.166	1.150	1.168	...	1.153	1.171	1.153	1.171
$P_C$	1.217	1.208	1.231	1.215	1.235	1.217	1.236		1.219	1.237	1.219	1.237
$P_A$	1.106	1.179	1.094	1.170	1.088	1.166	1.086		1.162	1.083	1.162	1.083
$P_B$	1.109	1.191	1.113	1.196	1.117	1.200	1.120	...	1.204	1.122	1.204	1.122
$P_C$	1.163	1.260	1.180	1.268	1.183	1.270	1.185		1.272	1.186	1.272	1.186
$P_A$	0.885	1.474	0.875	1.462	0.871	1.457	0.869		1.452	0.867	1.452	0.867
$P_B$	0.887	1.488	0.890	1.495	0.894	1.500	0.896	...	1.504	0.898	1.504	0.898
$P_C$	0.930	1.576	0.944	1.585	0.947	1.588	0.948		1.590	0.949	1.590	0.949

Table 2: Numerical simulation results.

## D Numerical Results with data of 23 OECD-countries

Difference	Mean	Std. Dev.	Min.	Max.
$\Delta_1 = P_{i100} - P_{i99}$	$-4.19 \cdot 10^{-8}$	$7.33 \cdot 10^{-8}$	$-2.38 \cdot 10^{-7}$	$1.19 \cdot 10^{-7}$
$\Delta_1 = P_{i100} - P_{i99}$	$-4.19 \cdot 10^{-8}$	$7.33 \cdot 10^{-8}$	$-2.38 \cdot 10^{-7}$	$1.19 \cdot 10^{-7}$
$\Delta_2 = P_{i100} - P_{i98}$	0	0	0	0
$\Delta_{10} = P_{i100} - P_{i90}$	0	0	0	0

Table 3: Amplitude and convergence of the simulation.

Value	Frequency	Percentage	Cumulation
$-2.38 \cdot 10^{-7}$	242	4.35	4.35
$-1.19 \cdot 10^{-7}$	1,650	29.64	33.99
0	3,498	62.85	96.84
$1.19 \cdot 10^{-7}$	176	3.16	100.00
Total	5,566	100.00	

Table 4: Descriptive statistics of  $\Delta_1 = P_{i100} - P_{i99}$ .

<b>Country</b>	<b>Min.</b>	<b>Mean</b>	<b>Max.</b>	<b>Std. Dev.</b>
Australia	1.444	1.483	1.519	0.022
Austria	1.371	1.399	1.424	0.017
Canada	1.002	1.041	1.123	0.035
Denmark	1.402	1.424	1.437	0.012
Finland	1.487	1.495	1.516	0.008
France	1.287	1.308	1.342	0.017
Germany	1.209	1.244	1.292	0.027
Greece	1.660	1.679	1.703	0.013
Hungary	1.451	1.510	1.632	0.061
Ireland	1.222	1.276	1.377	0.051
Italy	1.358	1.370	1.386	0.008
Japan	1.322	1.361	1.442	0.033
Korea	1.314	1.345	1.398	0.026
Netherlands	1.151	1.206	1.244	0.028
Norway	1.434	1.448	1.469	0.011
Poland	1.500	1.579	1.640	0.043
Portugal	1.623	1.637	1.651	0.008
Spain	1.418	1.442	1.479	0.018
Sweden	1.337	1.356	1.386	0.016
Switzerland	1.332	1.356	1.389	0.016
Turkey	1.551	1.613	1.703	0.047
United Kingdom	1.241	1.269	1.313	0.020
United States	1.442	1.506	1.542	0.028
<b>Total</b>	<b>1.002</b>	<b>1.406</b>	<b>1.703</b>	<b>0.150</b>

Table 5: Summary statistics of the multilateral resistances by country, over 11 years.

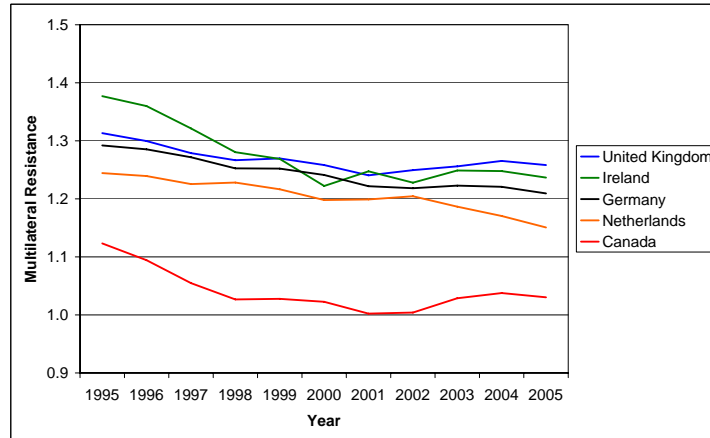


Figure 3: Countries with low Multilateral Resistances 1995 to 2005.

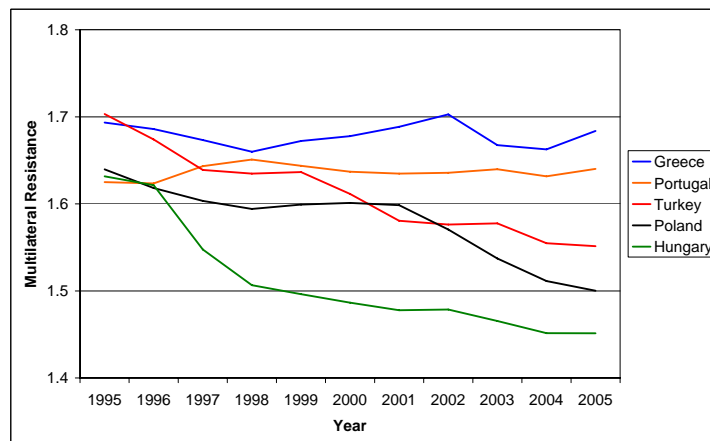


Figure 4: Countries with high Multilateral Resistances 1995 to 2005.



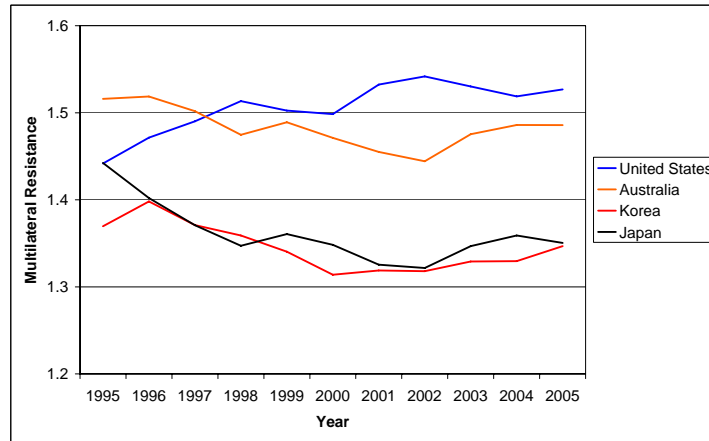


Figure 5: Multilateral Resistances of Australia, Japan, Korea, and the U.S., 1995 to 2005.

## E Estimation Results

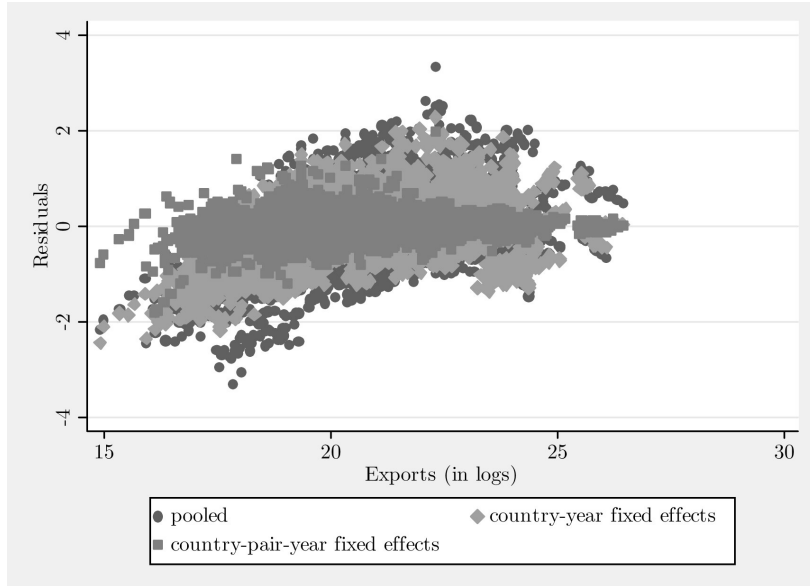


Figure 6: Basic Case – Residuals of the three panel specifications (without multilateral resistances)

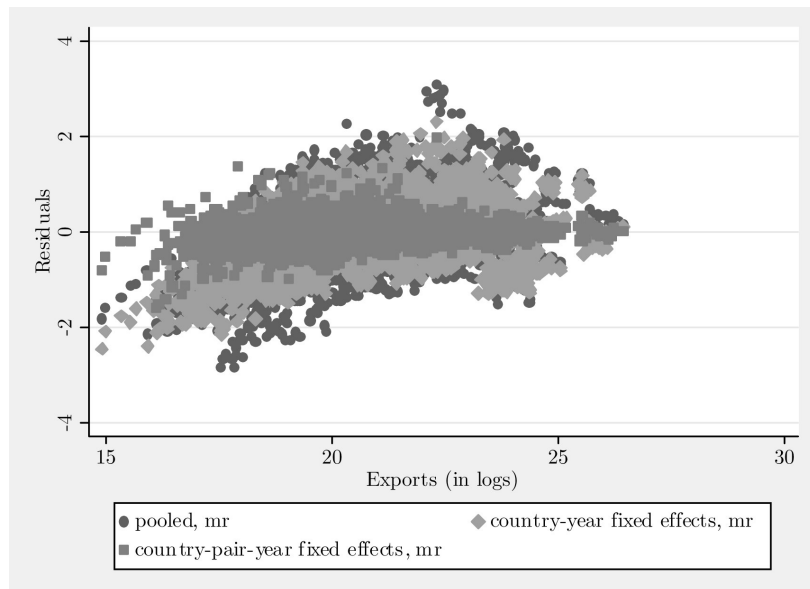


Figure 7: Basic Case – Residuals of the three panel specifications (with multilateral resistances)

	(1)	(2)	(3)	(4)	(5)	(6)
gdpi	0.864 (0.010)***	0.789 (0.010)***	0.368 (0.092)***	0.288 (0.094)***	0.310 (0.031)***	0.233 (0.030)***
gdpj	0.818 (0.010)***	0.817 (0.010)***	0.840 (0.106)***	0.808 (0.107)***	0.698 (0.031)***	0.672 (0.030)***
trfi	-0.734 (0.223)***	-0.816 (0.211)***	0.765 (0.218)***	0.798 (0.216)***	0.915 (0.075)***	0.651 (0.073)***
trfj	-0.378 (0.211)*	-0.511 (0.196)***	-0.020 (0.197)	-0.004 (0.198)	0.214 (0.075)***	-0.065 (0.073)
dist	-0.909 (0.015)***	-0.875 (0.015)***	-1.100 (0.024)***	-1.101 (0.024)***	—	—
lang	0.500 (0.045)***	0.452 (0.050)***	0.419 (0.041)***	0.414 (0.041)***	—	—
bor	0.322 (0.047)***	0.341 (0.046)***	0.115 (0.042)***	0.116 (0.042)***	—	—
exvol	-0.037 (0.010)***	-0.025 (0.008)***	-0.014 (0.007)**	-0.014 (0.007)**	-0.019 (0.004)***	-0.006 (0.004)
eui	0.088 (0.028)***	0.022 (0.027)	0.569 (0.079)***	0.397 (0.081)***	0.541 (0.028)***	0.398 (0.027)***
euj	0.066 (0.028)**	0.113 (0.028)***	0.084 (0.075)	0.032 (0.077)	0.072 (0.028)**	0.063 (0.027)**
landli	0.109 (0.034)***	-0.031 (0.033)	-2.511 (0.299)***	-1.282 (0.333)***	—	—
landlj	0.194 (0.036)***	0.169 (0.034)***	0.712 (0.190)***	0.229 (0.291)	—	—
isli	0.405 (0.031)***	0.514 (0.033)***	-0.013 (0.109)	1.147 (0.194)***	—	—
islj	0.018 (0.039)	0.006 (0.038)	-0.082 (0.117)	0.257 (0.193)	—	—
cwni	-0.389 (0.036)***	-0.820 (0.044)***	-1.464 (0.146)***	-1.698 (0.150)***	—	—
cwnj	0.282 (0.043)***	0.281 (0.045)***	-0.024 (0.289)	0.038 (0.290)	—	—
ebli	-0.503 (0.055)***	-0.327 (0.054)***	-2.701 (0.284)***	-1.045 (0.350)***	—	—
eblj	-0.152 (0.054)***	-0.163 (0.051)***	0.426 (0.156)***	0.451 (0.154)***	—	—
pi	—	-2.853 (0.132)***	—	-4.668 (0.627)***	—	-4.296 (0.212)***
pj	—	0.014 (0.123)	—	-1.358 (0.595)**	—	-0.604 (0.220)***
Const.	-12.435 (1.214)***	-8.738 (1.158)***	-5.274 (3.902)	-1.554 (3.961)	-11.407 (0.631)***	-4.550 (0.703)***
Obs.	4782	4782	4782	4782	4782	4782
R <sup>2</sup>	0.81	0.83	0.89	0.90	0.98	0.98

Robust standard errors in parentheses

\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%

Table 6: Basic Case – Estimates of the Standard Gravity Specification.

	(1)	(2)	(3)	(4)	(5)
gdpi	0.344 (0.022)***	0.284 (0.020)***	0.284 (0.020)***	0.351 (0.021)***	0.351 (0.021)***
gdpj	0.465 (0.022)***	0.566 (0.020)***	0.566 (0.020)***	0.482 (0.021)***	0.482 (0.021)***
tij	-5.982 (0.208)***	-7.293 (0.383)***	-7.293 (0.383)***	-7.219 (0.402)***	-7.219 (0.402)***
pi	—	-0.929 (0.271)***	-0.929 (0.271)***	—	—
pj	—	3.972 (0.280)***	3.972 (0.280)***	—	—
pp	—	—	—	1.402 (0.262)***	1.402 (0.262)***
Const.	3.741 (0.713)***	2.590 (0.591)***	2.590 (0.591)***	3.123 (0.620)***	3.123 (0.620)***
expij	-0.032 (0.002)***	-0.059 (0.002)***	-0.008 (0.003)***	-0.062 (0.002)***	-0.009 (0.003)***
trfi	-0.084 (0.008)***	-0.042 (0.007)***	-0.049 (0.008)***	-0.037 (0.007)***	-0.048 (0.008)***
trfj	-0.101 (0.008)***	-0.073 (0.007)***	-0.052 (0.008)***	-0.070 (0.007)***	-0.052 (0.008)***
exvol	0.003 (0.000)***	0.002 (0.000)***	0.001 (0.000)**	0.002 (0.000)***	0.001 (0.000)**
eui	-0.028 (0.003)***	-0.005 (0.003)*	-0.025 (0.003)***	-0.002 (0.003)	-0.024 (0.003)***
euj	-0.041 (0.003)***	-0.029 (0.002)***	-0.027 (0.003)***	-0.028 (0.002)***	-0.028 (0.003)***
pi	—	—	0.497 (0.026)***	—	—
pj	—	—	0.543 (0.024)***	—	—
pp	—	—	—	—	0.518 (0.016)***
Const.	2.283 (0.046)***	2.522 (0.040)***	1.045 (0.065)***	2.549 (0.040)***	1.052 (0.065)***
Obs.	4782	4782	4782	4782	4782

Standard errors in parentheses

\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%

Table 7: Simultaneous Equation Model – Country-pair-year fixed effects.

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